

Suggested Syllabus Math 290 A-B-C

A: Dynamical systems.

Introduction to dynamical systems. Definition and examples. Gradient flows and Hamiltonian systems. Existence and uniqueness of solution. Equilibria and periodic solutions. Introduction to bifurcation: Types of bifurcations and Exchange of Stability. Bifurcation of stationary solutions. Bifurcation of periodic solutions.

B: Perturbation and Asymptotic Methods.

Introduction to asymptotic approximations, examples of regular and singular perturbations. Initial value problems for ODEs. Methods of multiple scales and averaging. Boundary value problems for ODEs. The WKB method, turning points and matched asymptotic expansions. High frequency wave propagation. - Homogenization and effective medium theory.

C: Calculus of variations.

Basic problems in the calculus of variations and direct methods. Euler Lagrange equations. Theorems of DuBois-Reymond and Haar. Erdman's corner condition and Euler boundary conditions. The second variation and the Legendre condition. Weak and strong minima. The Hamiltonian and Hamilton-Jacobi equations. Variational problems with subsidiary conditions. Applications.

Part A:

Books:

[L] G. Looss & D. D. Joseph: Elementary Stability and Bifurcation Theory, Springer 1980.

[A] A. Amann: Ordinary Differential Equations, de Gruyter 1983.

Schedule:

Introduction to dynamical systems. Examples. (3 weeks) [A] Chapter 1,3 and Notes. Existence and uniqueness of solutions. (1 week) [A] Chapter 2.

Introduction to bifurcation: Types of bifurcations and exchange of stability (2 weeks) [L] Chapter2 I-III. Bifurcation of stationary solutions. Bifurcation of periodic solutions. (3 weeks) [L] Chapter2 IV-VI.

Part B:

Books:

[B-O] C. M. Bender & S. A. Orszag: Advanced Mathematical Methods for Scientists and Engineers, McGraw-Hill 1991.

[K-C] J. Kevorkian & J. D. Cole: Perturbation Methods in Applied Mathematics, Springer-Verlag, 1981.

[H] M. H. Holmes: Introduction to Perturbation Methods, Springer 1991.

Schedule:

Introduction to asymptotic approximations. Method of stationary phase. Regular and singular perturbations. (2 weeks) [H] Chapter 1 and [B-0] Chapter 1.

Method of multiple scales. Application to nonlinear oscillator. Method of averaging. Eigenvalue problems and turning points. (4 weeks) [K-C] Chapter 3

Wave propagation the WKB method. (2 weeks) Notes.

Homogenization. (1 week) [H] Chapter 5.

Part C:

Books:

[C-H] Courant and Hilbert: Methods of Mathematical Physics, Vol. I.

[W] F. Wan: Introduction to the calculus of variations and its applications. Chapman & Hall, 1995.

Schedule:

Basic problems in calculus of variations. Direct methods. (2 weeks) [W] chapter 1 and [C-H] chapter 4 section 1, 2.

Euler-Lagrange equations. Theorems of DuBois-Reymond and Haar. Erdmann's corner condition and Euler boundary conditions. (2 weeks) [W] chapter 2, 3 and [C-H] chapter 4 section 3, 5.

The second variation and the Legendre condition. Weak minimum and strong minimum. (3 weeks) [W] chapter 4, 5.

The Hamiltonian and Hamilton-Jacobi equations. (1 week) [W] chapter 6 and [C-H] chapter 4 section 10.

Variational problems with subsidiary conditions. (1 week) [W] chapter 10 and [C-H] chapter 4 section 7.