Math 218 Suggested Syllabus

Math 218 ABC is for general graudate students in mathematics. It is the prerequisite of Math 240ABC and Math 250ABC. It contains three parts:

1. Topics in elementary point set topology and algebraic topology
2. General manifold theory
3. Differential topology and basic differential geometry

Prerequisite: Math 205 and Math 206.

Textbooks:

1. "*Topology*", 2^{nd} edition, J. Munkres, Prentice-Hall;
2. "*Foundations of differentiable manifolds and Lie groups*", F. Warner, GTM 94, Springer-Verlag.

Reference Books:

1. "*Analysis on manifolds*", J. Munkres, Addison-Wesley Publishing Company;
2. "Differential topology", M. Hirsch, GTM 33, Springer-Verlag;
3. "*Morse theory*", J. Milnor, Princeton University Press;
4. "*Topology from the differentiable viewpoint*", J. Milnor, The University Press of Virginia.

1. Elementary Topology
(For Math 218 A, Fall quarter)

1. Metric spaces, topology defined by a metric
2. Completeness, Baire Category Theorem
3. Product of metric spaces, equivalent metrics
4. Compactness and sequential compactness, Lindelöf Theorem
5. Continuous maps between metric spaces, Arzela-Ascoli Theorem
6. General topological spaces, basic properties, subspaces, continuous maps and homeomorphisms
7. Base and subbase of topology, local base
8. The countability Axioms, Lindelöf Theorem

9. The Separation Axioms, Urysohn Lemma and Urysohn Metrization Theorem
10. Compactness
11. Product topology
12. Zorn Lemma, Tychonoff Theorem
13. Connected spaces and path-connected spaces
14. Quotient spaces, classification of surfaces
15. Homotopy of paths
16. The fundamental group
17. Covering spaces
18. Computing the fundamental group, Van Kampen Theorem
19. Applications: Brouwer Fixed Point Theorem (2-D), Borsuk-Ulam Theorem (2-D), Ham-Sandwich Theorem
20. Homotopy of maps

2. General Manifold Theory and Basic Differential topology and geometry
(For Math 218 B and C, Winter and Spring quarter)

1. Review of maps between Euclidean spaces, differentiability, chain rule, inverse and implicit function theorem, Riemann and Lebesgue integral
2. Paracompactness and local compactness
3. Manifolds in $\mathbb{R}^n$, abstract topological and differentiable manifolds
4. Submanifolds and manifolds with boundary
5. Differential maps, immersion and submersion
6. Tangent and cotangent spaces, linearization of differentiable maps
7. Partition of unity, embed a compact manifold into Euclidean spaces
8. Measure zero sets, Whitney Embedding Theorem
9. Regular values, preimage of regular values, Record Lemma
10. Sard Theorem
11. Introduction to vector bundles, tangent and cotanget bundles
12. Subbundle, Riemannian metrics, orthogonal complement bundle
13. Existence of tubular neighborhood
14. Homotopically approximate continuous maps and homotopies by smooth maps and homotopies

15. Transversality
16. Transversal approximation, transversal extension
17. Transversal approximation and transversal extension on manifolds with boundary
18. Transversal approximation and transversal extension with parameters
19. Sections of vector bundles, vector fields and differentiable forms
20. Tensor algebra, wedge product, exterior derivatives
21. Flow of a vector field, one-parameter group of diffeomorphisms
22. Lie derivatives
23. Basic Morse theory
24. Orientation of manifolds
25. Intergration of functions and forms on manifolds
26. Stokes Theorem
27. Intergration on Riemannian manifolds
28. Classification of one-dimensional manifolds, Brouwer Fixed Point Theorem
29. Mod 2 mapping degree and mod 2 winding number, Borsuk-Ulam Theorem
30. Mapping degree and winding number, Hopf Theorem
31. Selected topics on differential topology: local mapping degree, Leray formula, intersection number, index of singular points of vector fields, Poincaré-Hopf Theorem