

SUGGESTED SYLLABUS - MATH 280 A-B-C
Mathematical Logic

Possible Sources: Enderton, H.: A mathematical introduction to Logic (Models, compactness and completeness and Incompleteness and Goedel's Theorems) Jech, T: Set Theory (Basic set theory, Constructible sets) Devlin, K: Constructibility (Constructible sets) Chang, C and Keisler, J.: Model Theory (Models, compactness and completeness; Basic model theory) Soare, R: Recursively enumerable sets and degrees (Basic recursion theory)

Quarter I

Basic Set Theory:

Language of set theory

Zermelo-Fraenkel axioms

Natural numbers and Ordinal numbers

Well-founded relations

Transfinite induction and schema of definition by transfinite recursion.

Ordinal arithmetic

Mostowski collapsing theorem

Cardinal numbers and basic cardinal arithmetic.

Models, Compactness and Completeness:

Languages and structures

Satisfaction

Formal deduction

Henkin Models

Compactness Theorem

Goedel's Completeness Theorem

Quarter 2

Basic Model Theory:

Diagrams and models constructed from constants

Loewenheim-Skolem Theorems

Elementary Chains. (An application: Robinson Joint consistency Theorem)

Craig's Interpolation Theorem and Beth's Theorem.

Types and saturated models

Omitting types

\aleph_0 -categorical theories

Stability
Ultraproducts
Quantifier elimination

Incompleteness:

Peano Arithmetic
Primitive recursive functions
Representability of relations and functions in arithmetic
Goedel numbers
Formalizing deduction in arithmetic
Recursive decidability
Incompleteness and recursive undecidability of Peano arithmetic
Goedel's Second Incompleteness Theorem

Quarter 3

Recursion Theory:

Partial recursive functions
Universal functions and predicates
s-m-n theorem
Recursion Theorem
Many-one reducibility
Creative sets
Arithmetic hierarchy
The jump operator
Degrees of unsolvability
Priority method

Constructible Sets:

Relativization and absoluteness
The constructible hierarchy
Reflection Theorem
Zermelo-Fraenkel axioms in the constructible universe
Consistency of the Axiom of Choice and Generalized Continuum Hypothesis
Basic infinitary combinatorics in the constructible universe