Math 105A Final Exam Practice Exercises

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** Don't forget about Midterm material, too! (Refer to our old materials).

Reminder: Make sure you are comfortable with the Homework and Quiz Problems first!!
You must know Definitions, Theorems, and general algorithms, too.

1. Aside from HW 6: 7.1.12. (Check the three parts of the definition to be a norm.)

2. We mentioned it in discussion: 7.1.15, the strengthened version of Cauchy Schwarz.

3. 7.1.11. (A bit difficult)
If T is a (symmetric) Positive Definite matrix, show that $||\vec{x}||_T = \sqrt{\vec{x} \cdot T\vec{x}}$ is a norm on $\mathbb{R}^n$.
Hints: Use that a symmetric positive definite matrix has n real eigenvalues and an orthonormal basis of corresponding eigenvectors for everything.
For the Triangle Inequality: Square both sides. Apply Cauchy-Schwarz on terms that don't cancel.

4. 7.2.1,5,9. They're all related. It's more important to do the same parts (e.g. parts (a,b,d)).
At least the 2 by 2 matrices because the computations are easier.

5. 7.2.15b with 7.2.16. Show that $\rho(A) = ||A||_2$ when A is Symmetric.
Fact: A (real valued) symmetric matrix has only real eigenvalues.
(I also recommend the other parts of 7.2.15, too).

6. Aside from HW 7: Similar to 7.3.14, show that $||T_g||_\infty < 1$ with the Gauss Seidel iterative. (Did in Discussion, so review it.)

7. Aside from HW 8: 7.4.7b - Find the optimal choice of ω for 7.4.1b.

8. Aside from HW 8: 9.2.13 is fundamental. 9.2.16, and 18 are interesting.

9. From HW 9 on Power Method: 9.3.18a,b,d.

10. Aside from HW 9:
On shifting the QR algorithm. We discussed the Wilkinson Shift - the idea is to use a shift based on the eigenvalue of the bottom-right 2 by 2 matrix that is closest to the (n, n) entry.
(i) For a general 2×2 matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, find its eigenvalues.
(ii) Write a Matlab code that computes these eigenvalues and picks out the one closest to d.
Write this code by hand, and without using `eig(M)`. You can use `det(M)` and `trace(M)` if you wish.