Math 105A Summer 2017 Sample Questions (for Midterm)

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Problem 1

Let $g(x) = 2 - e^{-x}$. Show that fixed point iteration, $p_{n+1} = g(p_n)$ converges for any $p_0 \in [1, 3]$.

Problem 2

Find a fixed point iteration $p_{n+1} = g(p_n)$ converging quadratically for the equation $x = \cos(x)$ for sufficiently close starting points p_0. Include a justification for the quadratic convergence.

Problem 3

Suppose the sequence $\{p_n\}_{n=1}^{\infty}$ converges quadratically to $p = 0$. Let $r_n = p_{2n}$. What is the order of convergence of $\{r_n\}_{n=1}^{\infty}$?

Problem 4 (Example 2.5.8)

(a) Turn the root finding problem of solving $x - 2^{-x} = 0$ in $[0, 1]$, into a fixed point problem.
(b) Using (a) and $p_1 = 1$, find $p_1, ..., p_5$. Then find $\hat{p}_1, \hat{p}_2, \hat{p}_3$ by applying Aitken's Δ^2 method.
(c) How could we get a quadratic rate of convergence if we start p_1 sufficiently close?

Problem 5

Consider the following Matlab code to multiply two lower triangular matrices L_1, L_2 that are $n \times n$,

```
L =  zeros(n,n);
for i = 1:n
      for j = 1:i
            for k = j:i
                  L(i,j) = L(i,j) + L1(i,k)*L2(k,j);
            end
       end
end
```

Find the total operation count of the algorithm (that is, the number of additions and multiplications for this code). Finding the leading order term with correct coefficient is sufficient.

Problem 6

(a) Perform Gaussian Elimination with Partial Pivoting on the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 6 \\ -1 & 2 & 5 \end{bmatrix}$.
(b) Use the calculations in (a) to find the matrices P, L, U in the factorization $PA = LU$.

Problem 7

Find all values of α, β, if any, such that this matrix is diagonally dominant: $\begin{bmatrix} 3 & \beta & 1 \\ 2\alpha & 3 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

(Optional): What α, β would make it positive definite?