Solutions

Math 105A Quiz 4 - July 12

Please put name on front & ID on back for redistribution!

Show all of your work. *There is a question on the back side.*

1. [9pts] (a) Find the inverse of the matrix

* At Beginning: I said to defer to Partial Pivoting if scaled is inconclusive!

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

- Scales are circled in Red.
- Scales are across rows.

by using the *Gauss Jordan Method with Scaled Pivoting* to reduce $[A \vdots I] \to [I \vdots A^{-1}]$.

+2

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

Tie with a_{11}, a_{21} but $|a_{21}|$ larger.

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

+2 Swap.

$E_2 = E_2 - \frac{1}{2}E_1$.

$E_3 = E_3 - \frac{1}{2}E_1$.

$$\sim \left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 1 & 1 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 2 & 0 & -\frac{1}{2} & 1 \end{array}\right]$$

(No pivoting needed)

+2 not swapped.

$E_3 = E_3 + E_2$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 1 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 3 & 1 & -1 & 1 \end{array}\right]$$

Rescale pivots

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1/2 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{array}\right]$$

+3

$E_2 = E_2 + 2E_3$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1/2 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & -\frac{4}{3} & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & -\frac{1}{3} \\ 0 & 1 & 0 & -\frac{4}{3} & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{array}\right]$$

Last operation was $E_1 = E_1 - \frac{1}{2}E_2$.

+2

So $$\boxed{A^{-1} = \frac{1}{3}\begin{bmatrix} 2 & 1 & -1 \\ -4 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}}.$$

(solns)

(b) [3pts] Is the matrix from (a) indeed the inverse? Check by computing AA^{-1} or $A^{-1}A$. I am grading for a correct conclusion here, based on your answer in (a).

$$A^{-1}A = \frac{1}{3}\begin{bmatrix} 2 & 1 & -1 \\ -4 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 2+2-1 & 1-1 & 2-2 \\ -4+2+2 & 1+2 & -4+4 \\ 1-2+1 & -1+1 & 1+2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(+3) So yes it's indeed the inverse ✓ ($A^{-1}A = I$).

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \checkmark$$

2. [8pts] Prove that if A, B are nonsingular $n \times n$ matrices, then $(AB)^{-1} = B^{-1}A^{-1}$.

Sol 1: We need to verify 2 things.

- $(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1}$

  +4 $= AIA^{-1} = AA^{-1} = I$ ✓

- $(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B$

  +3 $= B^{-1}IB = B^{-1}B = I$ ✓

Since both orders yield Identity, this defines +1

$(AB)^{-1} = B^{-1}A^{-1}$ ▨

Sol 2: Just verify one,

ie. $(AB)B^{-1}A^{-1} = AIA^{-1}$

+4 $= AA^{-1} = I$ ✓

(+) Since inverses are unique, +3 we must have

$(AB)^{-1} = B^{-1}A^{-1}$.

+1 ▨

(conclusions are +1).