Math 2D Extra Practice Problems for MT

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*** DISCLAIMER *** Your primary study material should be your Homeworks! Make sure you have those down as they are directly the professor's practice problems he posted!

These problems are for if you get tired of your homework problems. They are from another professor's exams - Hamid Hezari. Some questions may be quite different or difficult compared to the homework. He asks a lot of insightful questions that have good concepts underlying them.

(10.1-3) Parametric and Polar Curves

1. Let $C(t) = (t^3 - 3t, t^2)$ parameterize a curve on $-\infty < t < \infty$.
(a) Find the points on the curve with vertical or horizontal tangent lines.
(b) Sketch the curve C by finding its x, y intercepts and by studying $\lim_{t\to\infty} c(t)$ and $\lim_{t\to-\infty} c(t)$.
Use arrows to indicate direction with increasing t. Show the vertical/horizontal tangents from (a) on your graph, too.

2. Let $C(t) = (e^t - 1,\ e^{2t})$ on $-\infty < t < \infty$.
(a) Compute the first and second derivatives, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.
(b) Find the equation of the tangent line at (0,1).
(b) Eliminate the parameter to find a Cartesian equation of the curve. Graph it, along with the tangent line from part (a).

3. Consider the polar curve $r = 2\theta$ where $0 \le \theta \le 8\pi$. Sketch this curve in the θr plane and the xy plane. Use arrows to indicate the direction of the curve.

4. Consider the polar curve $r = 2\sin\theta$ where $0 \le \theta \le \pi$.
(For practice, sketch this curve in the θr plane and the xy plane too, like in #3).
(a) Compute $\frac{dy}{dx}$ for the polar curve.
(b) Then, use the derivative to find the equation of the tangent line at $\theta = \pi/2$.

(12.1-6) Vectors, Equations of Lines and Planes, Quadric Surfaces

5. Let A be the point $(1, 1, -1)$ and P be the plane $x + 2y - z = 2$.
(a) Show that point A is not on the plane P. Then, find the parametric equation of the line going through A and is perpendicular to P.
(b) Find the equation of the plane that contains A and is parallel to P.
(c) *Optional!* Find the distance between point A and plane P.

6. Let A be the same point $(1, 1, -1)$ again, and let L be the parametric line

$$\vec{r}(t) = \langle t, 2t + 1, 2 - t\rangle.$$

(a) Show that A is not on the line L. Then, find the parametric equation of the line passing through A and parallel to L.

(b) Find the equation of the plane containing A and is perpendicular to L.
(c) *Optional!* Find the distance between point A and line L.

7. Consider the triangle T with vertices $A(1,1,1),\ B(2,2,0),\ C(0,2,3)$.
(a) Find the area of T.
(b) Find the equation of the plane containing the triangle.
(c) Find the length of the altitude of the triangle which is perpendicular to the base AB.

8. Consider a different triangle T where the vertices are $A(1,1,-1),\ B(2,1,0),\ C(0,2,1)$.
(For practice, do parts (a),(b) again from the last problem, #7).
Find the angle of vertex A of the triangle.

9. Some Quadric Surfaces:
(a) Consider $z = x^2 + \frac{y^2}{4}$. Use $z = k$ traces to sketch it. What kind of surface is it?
(b) Now consider $z^2 = x^2 + \frac{y^2}{4}$ for $z \geq 0$. What kind of surface is this one? Also sketch it.
(c) Although the traces look similar for both, the two surfaces are different. Explain the difference by considering the $x = 0$ trace for each surface.
(d) A separate problem: What kind of surface is $z - 1 = y^2 - x^2 - 2y$? Sketch it.

(13.1-4, 14.1) Vector Functions, Functions of 2 or More Variables

10. Let $\vec{r}(t) = \langle \sin t, \sin t, 2\cos t \rangle$ for $t \in \mathbb{R}$.
(a) Show/Verify that this curve lies on the surface of an ellipsoid and find the equation of the ellipsoid. Also, show that the curve C lies on a plane and find the equation of the plane.
(b) Sketch the ellipsoid, plane, and curve all in one graph. Use arrows to indicate the direction of the curve with increasing t.
(c) What is $\vec{v}(t)$? Find $\vec{r}(\pi/4)$ and $\vec{v}(\pi/4)$ and plot it on the graph in (b).
(d) Find the parametric equation of the tangent line to the curve at $\vec{r}(\pi/4)$.

11. Let $\vec{r}(t) = \langle 2\sin t, t^3, 2\cos t \rangle$ on $-\infty < t < \infty$.
(a) Show/Verify that the curve sits on the surface of a cylinder, and find the cylinder's equation.
(b) Find the velocity vector $\vec{v}(0)$ at the point $\vec{r}(0)$. (This is the direction of the curve). Also, find the equation of the tangent line to the curve at this time and point.
(c) Sketch the curve, the cylinder, the point $\vec{r}(0)$, and the velocity vector $\vec{v}(0)$ all in one graph.

12. Conisder $f(x,y) = \frac{1}{1+x^2+y^2}$. Sketch the level curves $f(x,y) = c$ for $c = \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$.
What is the domain of validity for f?

13. Sketch the level curves of the function $f(x,y) = e^{x^2+y^2}$ for $c = 1, 2, 3, 4$.

14. Sketch the level curves of the function $f(x,y) = \frac{x-y}{x+y}$ for $c = -1, 0, 1, 2$.
What is the domain of validity for this function?