Solns

(Evening)

# Math 2D Quiz 2 - September 29th
## Please put ID on back for redistribution!

Show all of your work. *There is a question on the back side.*

**1.** Let $x = t^2$, $y = \ln t$ parameterize a curve for $0 < t < \infty$.

a) [4pts] Plot the curve. Indicate with arrows the direction the curve is traced as $t$ increases.

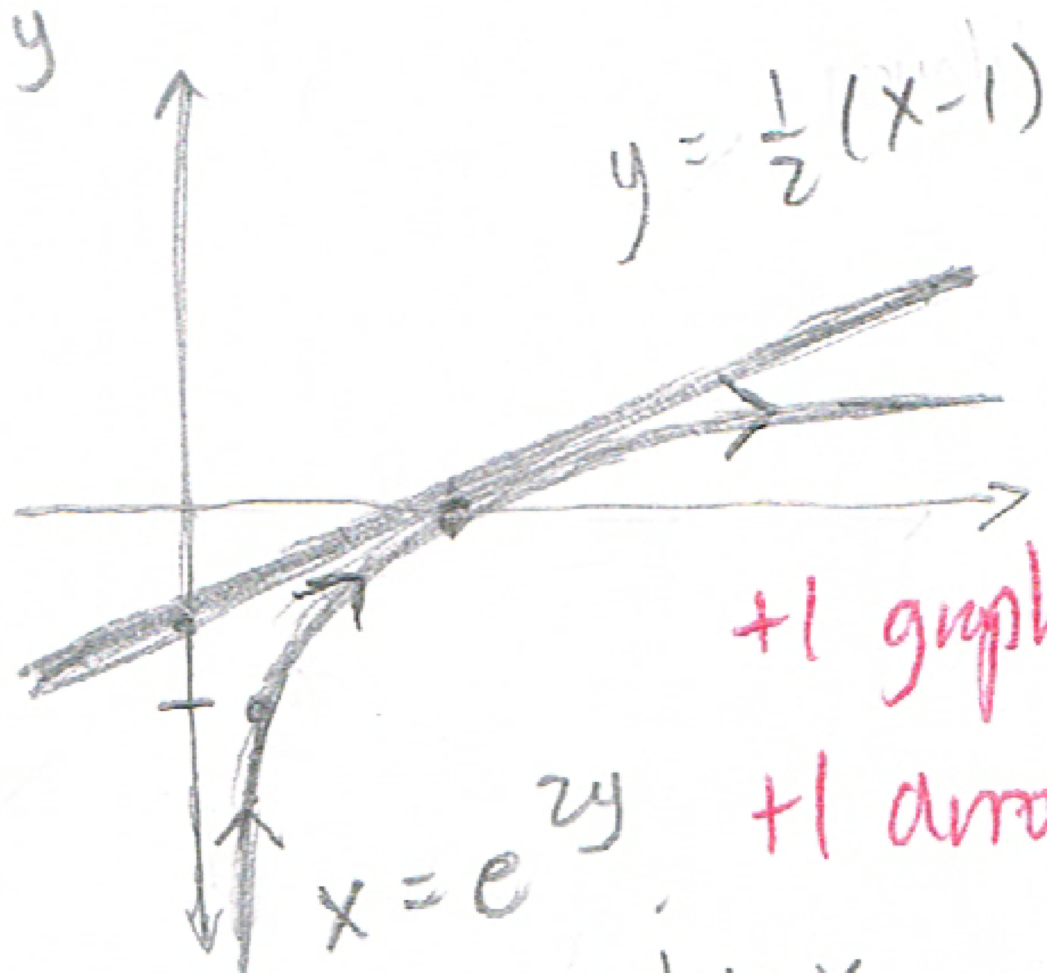
- $x = t^2 \Rightarrow t = \sqrt{x}$, $y = \frac{1}{2}\ln x$

(pull down 1/2 power to front)

$\boxed{\text{or}}$

- $y = \ln t \Rightarrow t = e^y$, $x = e^{2y}$.

+2




ie. $y = \frac{1}{2}\ln x$.

+1 graph

+1 arrow

or,

| $t$ | $x$ | $y$ |
|---|---|---|
| "0" | 0 | $-\infty$ |
| $1/e$ | $1/e^2$ | $-1$ |
| 1 | 1 | 0 |
| $e$ | $e^2$ | 1 |

(+1 table
+3 graph)

b) [6pts] Find the equation of the tangent line to the curve at the time $t = 1$. Draw the tangent line onto your graph in (a). Does it look sensible? (Hopefully!)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1/t}{2t} = \frac{1}{2t^2}. \quad +2$$

At $t=1$: $(x,y) = (1,0)$ +1

$$\left.\frac{dy}{dx}\right|_{t=1} = \frac{1}{2}. \quad +1$$

Thus, the eqn is $\boxed{y - 0 = \frac{1}{2}(x-1),}$

ie. $\boxed{y = \frac{1}{2}(x-1)}$ +1

+1 Graphed.

Solns

**2.** Let $x = \sin t$, $y = \cos t$ parameterize a curve for $0 \le t \le \pi$.

a) [4pts] Find the slope of the tangent line to the curve as a function of $t$.
At what $t$-values in the given interval is the tangent horizontal? What about vertical?

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-\sin t}{\cos t} = \boxed{-\tan(t).} \quad +2$$

Horizontal: $\frac{dy}{dx}$ is $0 \Rightarrow \sin t = 0$, $\boxed{t = 0, \pi}$ $+1$

Vertical: $\frac{dy}{dx}$ is $\pm\infty$ (division by 0) $\Rightarrow \cos t = 0$, $\boxed{t = \frac{\pi}{2}}$ $+1$

b) [4pts] Compute the second derivative, $\frac{d^2y}{dx^2}$.
On the interval given, when is the curve is concave up? When is it concave down?

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt} = \frac{-\frac{d}{dt}(\tan(t))}{\cos t} = \boxed{\frac{-\sec^2 t}{\cos t}.} \quad +2$$

- $\sec^2 t \geq 0$ b/c squared, so sign of $\frac{d^2y}{dx^2}$ depends on $\cos(t)$.

↳ On $(0, \pi)$, $\cos t > 0$ on $(0, \frac{\pi}{2})$, $\cos t < 0$ on $(\frac{\pi}{2}, \pi)$.

Thus: Concave Up: $(\frac{\pi}{2}, \pi)$ $+1$ Concave Down: $(0, \pi/2)$ $+1$

(careful of sign of $\frac{d^2y}{dx^2}$!) b/c $d^2y/dx^2 = - \frac{\sec^2 t}{\cos t}$.

c) [2pts] Set up, but you do not need to evaluate, the integral that finds the arclength.

$+2$

$$L = \int_0^{\pi} \sqrt{\cos^2 t + \sin^2 t}\, dt = \int_0^{\pi} 1\, dt = \pi.$$

↑ can reuse from (a).