Solutions

Name & ID

# Math 2D Quiz 1 Morning - September 29th

**Please put ID on back for redistribution!**

Show all of your work. *There is a question on the back side.*

1. Let $x = e^t$, $y = e^{-2t}$ parameterize a curve for $-\infty < t < \infty$.

a) [4pts] Plot the curve. Indicate with arrows the direction the curve is traced as $t$ increases.

(+2)

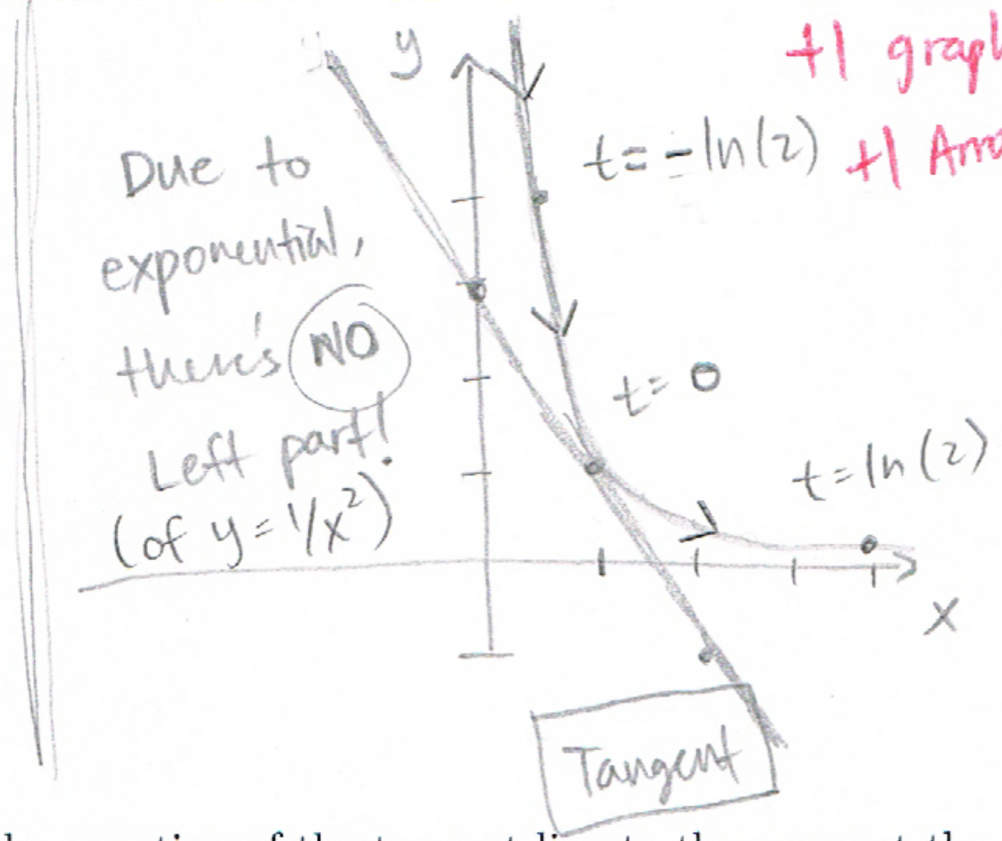
Eliminate Parameter:

$y = \frac{1}{e^{2t}}$ and $x = e^t$

so $\boxed{y = 1/x^2}$

Alternatively, $e^{-2t} = y$ gives

$e^t = \frac{1}{\sqrt{y}}$, $x = \frac{1}{\sqrt{y}}$.




Due to exponential, there's NO Left part! (of $y = 1/x^2$)

+1 graph

+1 Arrow

Or, table of pts:

| t | x | y |
|---|---|---|
| -3 | $e^{-3}$ | $e^6$ |
| -2 | $e^{-2}$ | $e^4$ |
| -1 | $e^{-1}$ | $e^2$ |
| 0 | 1 | 1 |
| 1 | e | $e^{-2}$ |
| 2 | $e^2$ | $e^{-4}$ |
| 3 | $e^3$ | $e^{-6}$ |

(+1)

I prefer: use ln,

| t | x | y |
|---|---|---|
| $-\ln 3$ | 1/3 | 9 |
| $-\ln 2$ | 1/2 | 4 |
| 0 | 1 | 1 |
| $\ln 2$ | 2 | 1/4 |
| $\ln 3$ | 3 | 1/9 |

Add Graph B 3pts.

b) [6pts] Find the equation of the tangent line to the curve at the time $t = 0$. Draw the tangent line onto your graph in (a). Does it look sensible? (Hopefully!)

- $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2e^{-2t}}{e^t} = -2e^{-3t}$. +2
- At $t=0 \rightsquigarrow (x,y)\big|_{t=0} = (1,1)$. +1
  $\rightsquigarrow \frac{dy}{dx}\big|_{t=0} = -2$. +1

Thus, Tangent Line Eqn: $\boxed{y - 1 = -2(x-1)}$ +1

(same as $\boxed{y = -2x + 3}$)

(+1)

↳ Drawn, looks like it's tangent! ☺

(ID)

**2.** Let $x = t^2 + 1,\ y = t^2 + t$ parameterize a curve for $-\infty < t < \infty$.

a) [4pts] Find the slope of the tangent line to the curve as a function of $t$.
At what $t$-values is the tangent horizontal? What about vertical?

Slope $= \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \boxed{\frac{2t+1}{2t}}$ +2 (which equals $1 + \frac{1}{2t}$)

Horizontal: Slope $= 0 \rightsquigarrow \boxed{t = -1/2}$ +1

Vertical: Slope $\rightsquigarrow \pm\infty$, $\boxed{t = 0}$ +1

b) [4pts] Compute the second derivative, $\frac{d^2y}{dx^2}$.
On the given interval, when is the curve is concave up? When is it concave down?

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt} = \frac{\frac{d}{dt}\left(1 + \frac{1}{2t}\right)}{2t}$ ← Easier to take derivative.

$= \frac{-\frac{1}{2t^2}}{2t} = \boxed{-\frac{1}{4t^3}}$ +2

Concave Up: $\frac{d^2y}{dx^2} = -\frac{1}{4t^3} > 0$ needed, so $\boxed{t < 0}$ +1

Concave Down: Now need $-\frac{1}{4t^3} < 0$ so $\boxed{t > 0}$ +1

c) [2pts] Set up, but do not evaluate, the integral that finds the arclength of the curve. (on $[0, 100]$)

↳ $L = \int_0^{100} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ +1

$= \int_0^{100} \sqrt{4t^2 + (2t+1)^2}\, dt = \int_0^{100} \sqrt{8t^2 + 4t + 1}\, dt$ +0!