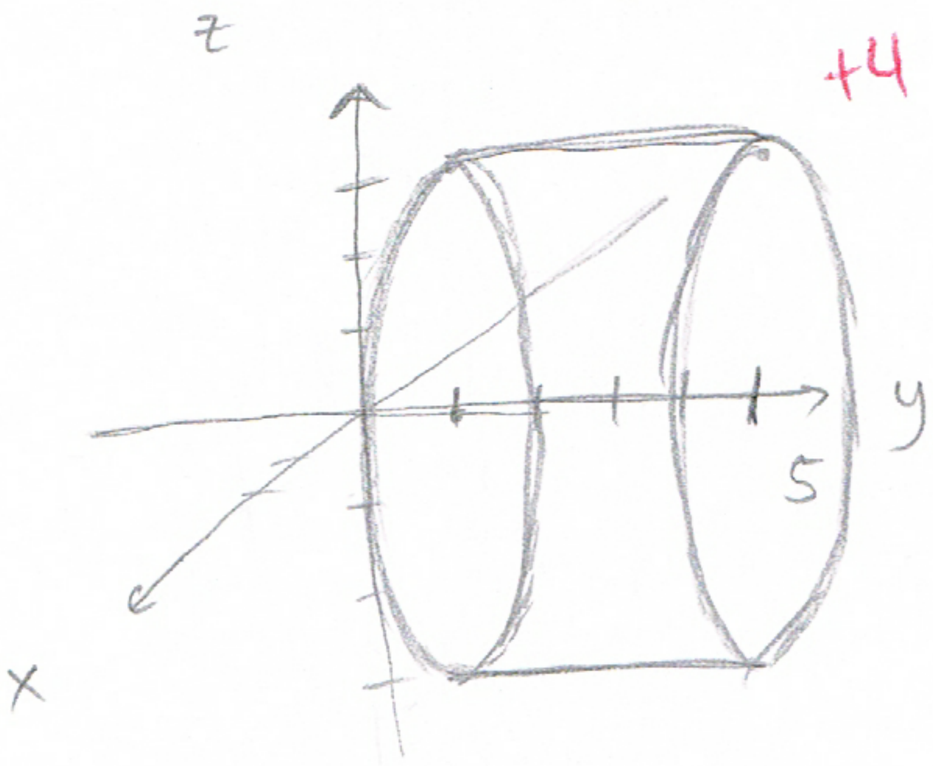
Solutions

Math 2D (Actual) Quiz 2 Evening - October 6th

Please put ID on back for redistribution! (And name on front).

Show all of your work. *There is a question on the back side.*

1. (a) [4pts] Draw the region of $\mathbb{R}^3$ given by: $\frac{x^2}{4} + \frac{z^2}{9} = 1, \quad 1 \leq y \leq 5$.
If you cannot draw this, describing it correctly will get a little partial credit. (+1)

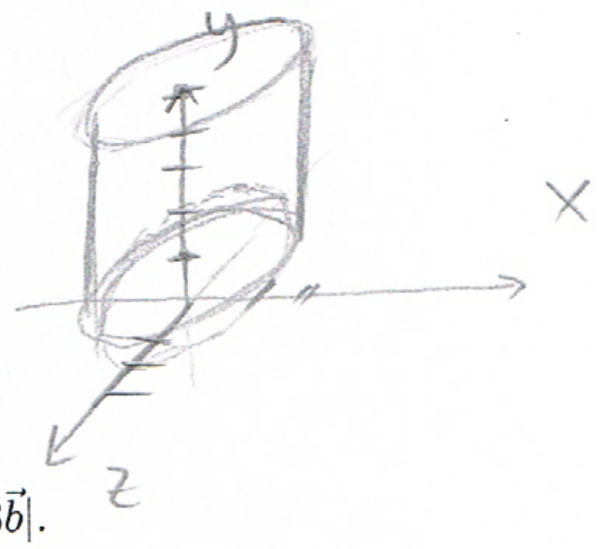


+4

Boundary of the cylinder from y=1 to 5.

Cross Section is an ellipse.

Alt:



(b) [4pts] Let $\vec{a} = \langle 8, 1, -2 \rangle, \quad \vec{b} = \langle 5, -2, 1 \rangle$. Compute $2\vec{a} - 3\vec{b}$ and also $|2\vec{a} - 3\vec{b}|$.

- $2\vec{a} - 3\vec{b} = 2\langle 8, 1, -2 \rangle - 3\langle 5, -2, 1 \rangle$

$= \boxed{\langle 1, 8, -7 \rangle}$ +2

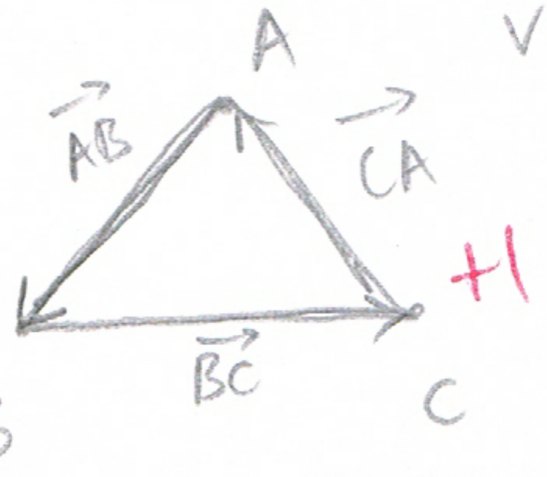
- $|2\vec{a} - 3\vec{b}| = \sqrt{1 + 64 + 49}$

$= \boxed{\sqrt{114}}$ +2

(c) [2pts] Let the vertices of a triangle be points A, B, C. What is $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$? Briefly explain/prove your answer. (You may prove it graphically/geometrically).

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$. +1

Pf: For a triangle with Tip-Tail Vector Addition we start and end at same place so overall, we never moved.



+1

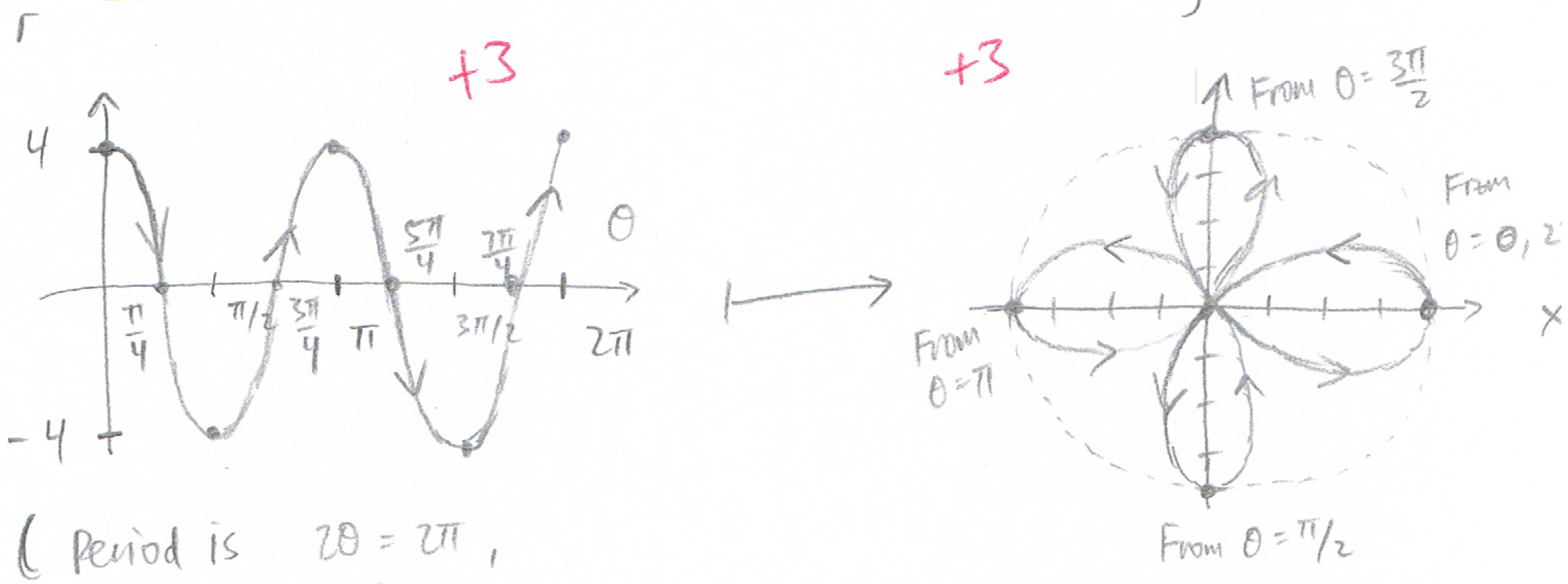
Alt: Note $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ and $\overrightarrow{AC} = \ominus\overrightarrow{CA}$!

Twice as oscillatory!

Solns

2. Let $r = 4\cos(2\theta)$ where $0 \le \theta \le 2\pi$.

(a) [6pts] Plot the curve on the $r\theta$-plane first. Use that to plot the curve on the xy-plane. Indicate with arrows the direction the curves are traced as θ increases from 0.



(Period is $2\theta = 2\pi$, $\theta = \pi$)

(b) [4pts] Find the equation of the tangent line at $\theta = 3\pi/4$. (Here, $(x, y) = (0, 0)$).

$r = 4\cos 2\theta \Rightarrow$ $x = 4\cos 2\theta \cos\theta$, $y = 4\cos 2\theta \sin\theta$ (from $x = r\cos\theta$, $y = r\sin\theta$)

$$\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-8\sin 2\theta \sin\theta + 4\cos 2\theta \cos\theta}{-8\sin 2\theta \cos\theta - 4\cos 2\theta \sin\theta} \quad +2$$

$$\text{At } \frac{3\pi}{4}, \quad \left.\frac{dy}{dx}\right|_{\frac{3\pi}{4}} = \frac{-8\cdot(-1)\cdot\left(+\frac{\sqrt{2}}{2}\right) - 0}{-8\cdot(-1)\cdot\left(-\frac{\sqrt{2}}{2}\right) + 0}$$

$$= \ominus \frac{4\sqrt{2}}{4\sqrt{2}} = \boxed{-1} \quad +1$$

Thus, tan line is $\boxed{y = -x}$ +1