Solutions

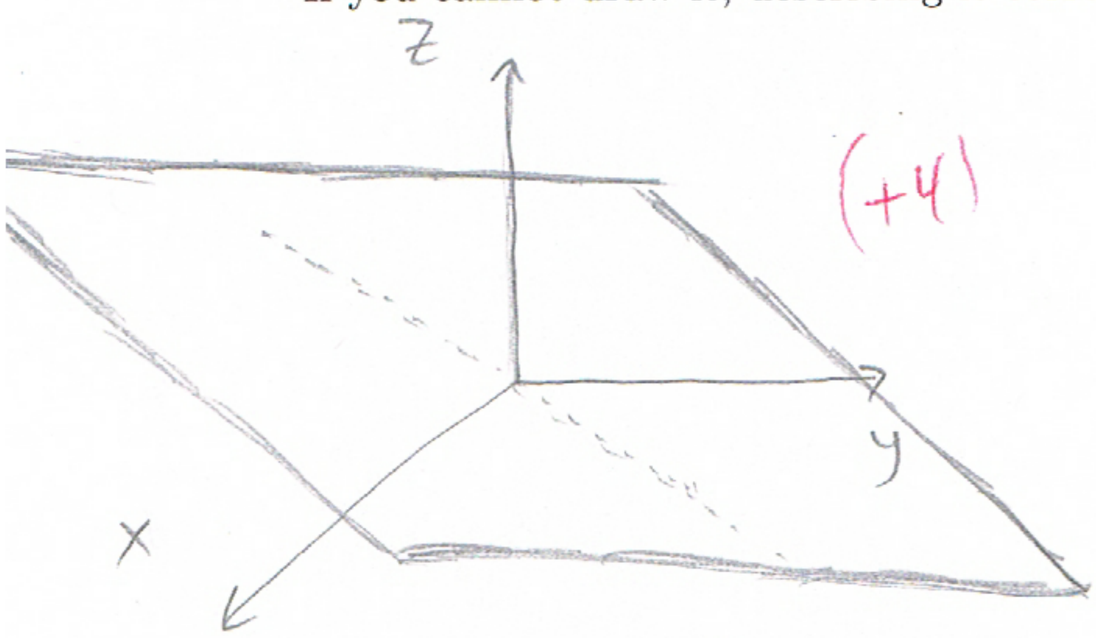
Math 2D Quiz 2 Morning - October 6th

Please put name on front & ID on back for redistribution!

Show all of your work. *There is a question on the back side.*

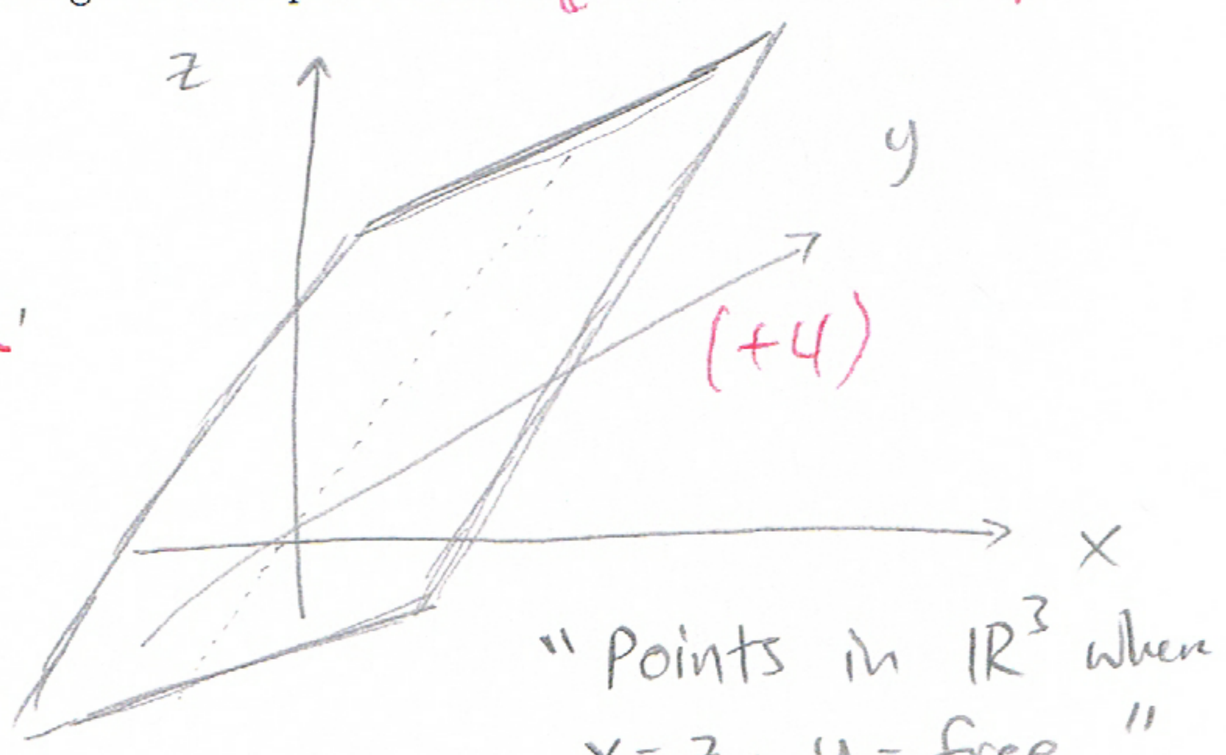
1. (a) [4 pts] Graph $x = z$ in 3-dimensional space, $\mathbb{R}^3$.

If you cannot draw it, describing it correctly will get a little partial credit. (+1) if bad graph.



(+4)

or,



(+4)

(Plane in $\mathbb{R}^3$)

"Points in $\mathbb{R}^3$ when $x=z$, $y=$ free."

(b) [4pts] Let $\vec{a} = 2\hat{i} - 3\hat{j}$, $\vec{b} = 4\hat{i} - 3\hat{j} + \hat{k}$. Compute $4\vec{a} - 3\vec{b}$. Also compute $|4\vec{a} - 3\vec{b}|$.

$$4\vec{a} - 3\vec{b} = 4(2\hat{i} - 3\hat{j}) - 3(4\hat{i} - 3\hat{j} + \hat{k})$$

$$= \boxed{-4\hat{i} - 3\hat{j} - 3\hat{k}} \quad +2$$

$$|4\vec{a} - 3\vec{b}| = \sqrt{16 + 9 + 9} = \boxed{\sqrt{34}} \quad +2$$

(c) [2pts] Find a vector with the same direction as vector $\vec{u} = \langle 8, 6, 0 \rangle$ with length L.

$$\hat{u} = \frac{\vec{u}}{|\vec{u}|} = \frac{1}{\sqrt{64+36}} \langle 8, 6, 0 \rangle$$

$$= \boxed{\frac{1}{10} \langle 8, 6, 0 \rangle = \langle \frac{4}{5}, \frac{3}{5}, 0 \rangle}$$ +1

So, we use

$$\boxed{\frac{L}{10} \langle 8, 6, 0 \rangle}$$

$(L\hat{u})$ +1 is the vecto

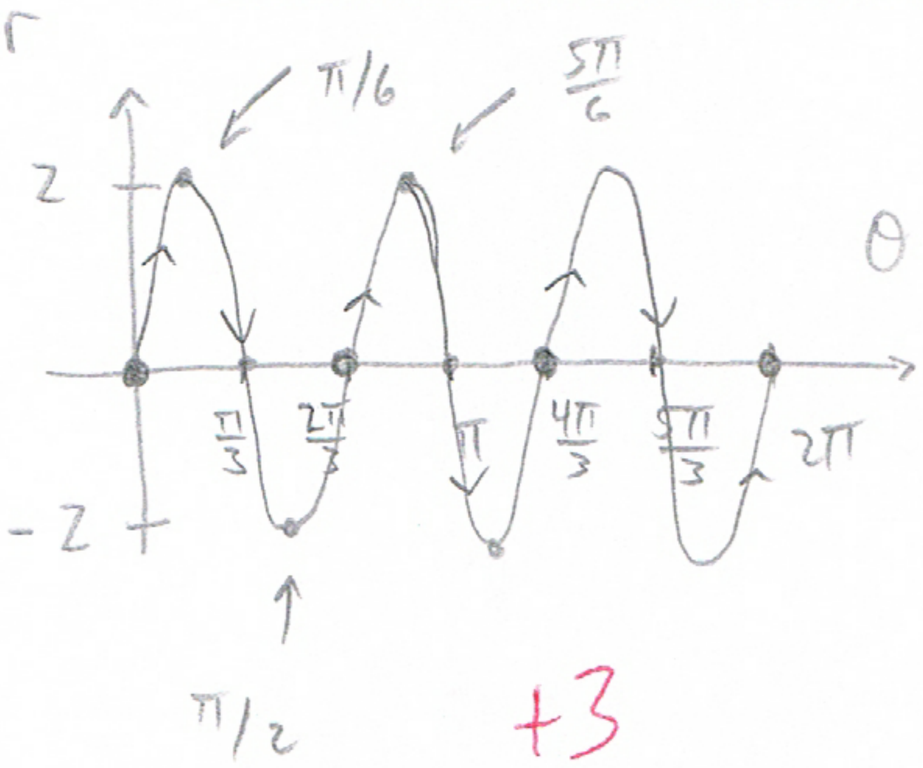
Three times as oscillatory.

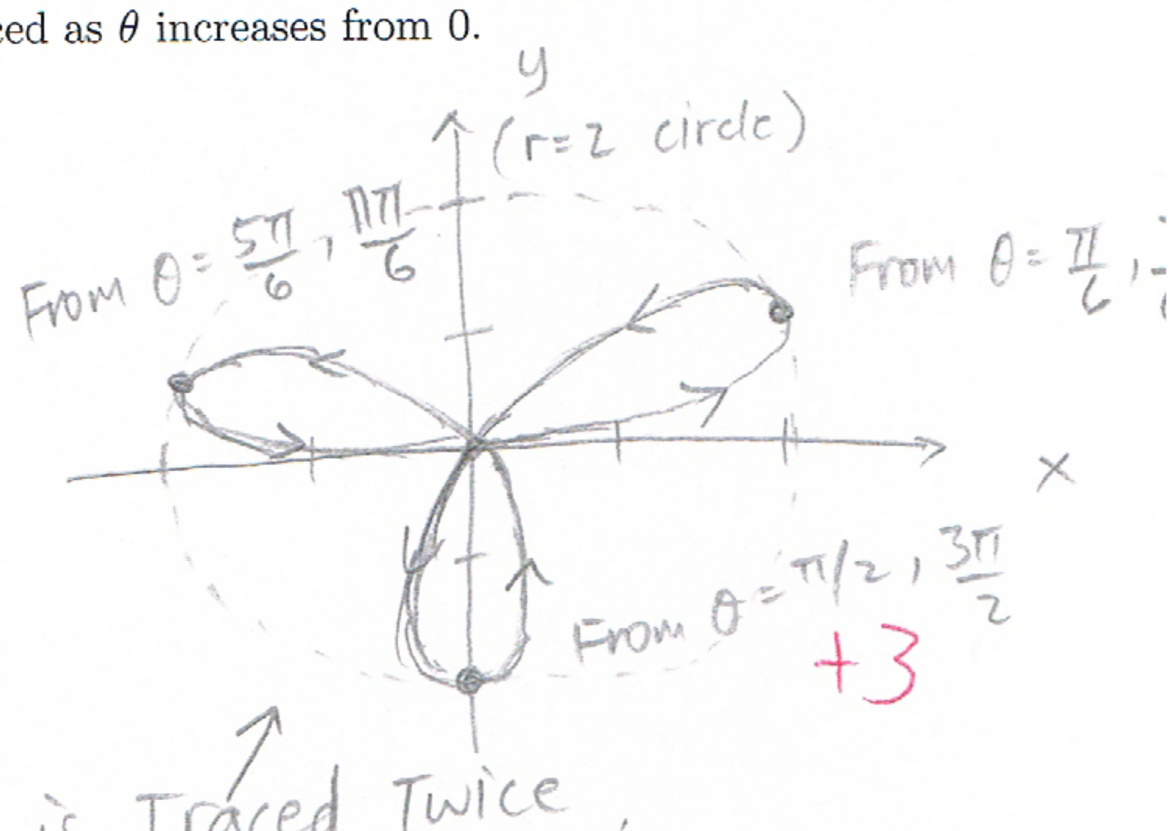
Solus

↙ Period: $3\theta = 2\pi$, $\theta = \frac{2\pi}{3}$

2. Let $r = 2\sin(3\theta)$ where $0 \le \theta \le 2\pi$.

(a) [6pts] Plot this curve on the $r\theta$-plane first. Use it to graph the curve on the xy-plane. Indicate with arrows the direction the curves are traced as θ increases from 0.





This is Traced Twice,
1st from $0 \le \theta \le \pi$,
and repeats from $\pi \le \theta \le 2\pi$

(b) [4pts] Find the equation of the tangent line at $\theta = \pi/3$. (Here, $(x, y) = (0, 0)$).

$x = r\cos\theta = 2\sin(3\theta)\cos\theta$

$y = r\sin\theta = 2\sin(3\theta)\sin\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2\cos\theta\sin 3\theta + 6\cos 3\theta\sin\theta}{-2\sin\theta\sin 3\theta + 6\cos 3\theta\cos\theta}$$ +2 (correct defn & derivative)

$$\text{At } \frac{\pi}{3},\ \left.\frac{dy}{dx}\right|_{\pi/3} = \frac{0 + 6\cdot(-1)\cdot\frac{\sqrt{3}}{2}}{0 + 6\cdot(-1)\cdot 1/2} = \frac{-3\sqrt{3}}{-3}$$

$= \boxed{\sqrt{3}}$ +1

Thus, Line is $\boxed{y = \sqrt{3}\,x}$ +1