Solutions

Math 2D Quiz 3 Evening - October 13th

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Show all of your work. *There is a question on the back side.*

1. (a) [5pts] Let $\vec{a}, \vec{b}$ be two vectors in $\mathbb{R}^3$. Define $Orth_{\vec{a}}\, \vec{b}$, by

$$Orth_{\vec{a}}\, \vec{b} = \vec{b} - Proj_{\vec{a}}\, \vec{b}.$$

Show that $\vec{a} \bullet Orth_{\vec{a}}\, \vec{b} = 0$ and that $\vec{a} \times Orth_{\vec{a}}\, \vec{b} = \vec{a} \times \vec{b}$.

+1

Here, $\mathrm{Orth}_{\vec{a}}\, \vec{b} = \vec{b} - \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{a}$.

+1

- $\vec{a} \cdot \mathrm{orth}_{\vec{a}}\, \vec{b} = \vec{a}\cdot\vec{b} - \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)(\vec{a}\cdot\vec{a})$

  $= \vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{b}\left(\frac{\cancel{|\vec{a}|^2}}{\cancel{|\vec{a}|^2}}\right) = 0$ ✓ +1

- $\vec{a} \times \mathrm{orth}_{\vec{a}}\, \vec{b} = \vec{a}\times\vec{b} - \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\underbrace{(\vec{a}\times\vec{a})}_{\text{zero!}}$ +1

  $= \vec{a}\times\vec{b}$ ✓

  +1

(b) [5pts] Determine if the following four points lie on the same plane:

$$A = (1,2,3),\ B = (2,3,6),\ C = (-1,5,-3),\ D = (-3,0,-9).$$

(+5)

(You may have observed all 4 pts lie on the $z = 3x$ plane)

Soln: Consider Parallelepiped w/ vertices $\vec{AB} = \langle 1,1,3\rangle$

$\vec{AC} = \langle -2,3,-6\rangle$ +2

$\vec{AD} = \langle -4,-2,-12\rangle$

could take any scalar triple product of the three

Then $\text{Volume} = |\vec{AD}\cdot(\vec{AB}\times\vec{AC})| = \left|\langle -4,-2,-12\rangle \cdot \det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 3 \\ -2 & 3 & -6 \end{bmatrix}\right|$

$= \left|\langle -4,-2,-12\rangle \cdot \left\langle \det\begin{bmatrix} 1 & 3 \\ 3 & -6 \end{bmatrix}, -\det\begin{bmatrix} 1 & 3 \\ -2 & -6 \end{bmatrix}, \det\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}\right\rangle\right|$

$= |\langle -4,-2,-12\rangle \cdot \langle -15, 0, 5\rangle| = |60 - 60| = \boxed{0}$ +2 the computing

Since Volume = 0 $\Rightarrow$ $\boxed{\text{They Lie on same plane}}$ +1

(solus)

"A" "B"

2. (a) [6pts] Find the equation of the plane which passes through the points $(0, -2, 5)$, $(-1, 3, 1)$ and is perpendicular to the plane $2z = 5x + 4y$.
We say plane 1 is perpendicular to plane 2 if plane 1 contains the normal vector of plane 2. +1

Sol: ↳ So, for plane 2, $5x + 4y - 2z = 0$, its normal $\boxed{\langle 5, 4, -2 \rangle}$ is a vector in our plane. We need another vector in our plane.

+1

From the 2 pts given ⇝ $\vec{AB} = \boxed{\langle -1, 5, -4 \rangle}$ is also on our plane.

Thus, $\vec{n} = \langle 5, 4, -2 \rangle \times \langle -1, 5, -4 \rangle = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 4 & -2 \\ -1 & 5 & -4 \end{bmatrix}$.

$= \langle \det \begin{bmatrix} 4 & -2 \\ 5 & -4 \end{bmatrix}, \ominus \det \begin{bmatrix} 5 & -2 \\ -1 & -4 \end{bmatrix}, \det \begin{bmatrix} 5 & 4 \\ -1 & 5 \end{bmatrix} \rangle$

$= \langle -6, 22, 29 \rangle$ +2

Using point B ⇝ Eqn is $-6(x+1) + 22(y-3) + 29(z-1) = 0$

Using point A ⇝ $-6x + 22(y+2) + 29(z-5) = 0$ +2

If you expanded, ⇝ $-6x + 22y + 29z = 101$ (Any is ok)

(b) [4pts] Find the equation of the line through the point (1,0,6) which is also perpendicular to the plane $x + 3y + z = 5$.

+2

↳ $\vec{n} = \langle 1, 3, 1 \rangle$ so this is the line's direction

So, our line eqn is $\vec{r}(t) = \langle 1, 0, 6 \rangle + t \langle 1, 3, 1 \rangle$

or, $x = 1 + t$, $y = 3t$, $z = 6 + t$ +2

or, $x - 1 = \dfrac{y}{3} = z - 6$ (Any is ok)