Solutions

Math 2D Quiz 3 Morning - October 13th

Please put name on front & ID on back for redistribution!

Show all of your work. *There is a question on the back side.*

1. (a) [5pts] Let $\vec{a} = \langle 1, 2, 2\rangle$, $\vec{b} = \langle 3, 0, 6\rangle$. Compute $Orth_{\vec{a}}\,\vec{b}$ where $Orth_{\vec{a}}\,\vec{b} = \vec{b} - Proj_{\vec{a}}\,\vec{b}$. Is $Orth_{\vec{a}}\,\vec{b}$ perpendicular to the vector $\vec{a}$? (Justify with dot product)

$$Orth_{\vec{a}}\vec{b} = \vec{b} - \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{a} \quad (+1)$$

$$\frac{15}{9} = \frac{5}{3}$$

$$= \langle 3,0,6\rangle - \left(\frac{3+12}{4+4+1}\right)\langle 1,2,2\rangle \qquad +2$$

$$= \langle 3,0,6\rangle - \frac{5}{3}\langle 1,2,2\rangle = \boxed{\left\langle \frac{4}{3}, -\frac{10}{3}, \frac{8}{3}\right\rangle}$$

$$\vec{a}\cdot Orth_{\vec{a}}\vec{b} = \langle 1,2,2\rangle \cdot \left\langle \frac{4}{3}, -\frac{10}{3}, \frac{8}{3}\right\rangle$$

$$= \frac{4}{3} - \frac{20}{3} + \frac{16}{3} = \boxed{0} \quad \text{so } \vec{a} \perp Orth_{\vec{a}}\vec{b}.$$

+2

(b) [5pts] Suppose we know that $\vec{u} \bullet \vec{w} = D$ where $D \geq 0$, and also that $\vec{u}\times\vec{w} = \langle a, b, c\rangle$. (Assume that the vectors $\vec{u}, \vec{w}$ are both nonzero).
Show that the angle between the two vectors is given by

$$\theta = \arctan\left(\frac{\sqrt{a^2+b^2+c^2}}{D}\right).$$

(Remark: If we had $D < 0$, we would have needed π + the arctangent). +1

Recall: $|\vec{u}\times\vec{w}| = |\vec{u}||\vec{w}|\sin\theta$ +1 and $\vec{u}\cdot\vec{w} = |\vec{u}||\vec{w}|\cos\theta$.

Here, $\sqrt{a^2+b^2+c^2} = |\vec{u}||\vec{w}|\sin\theta$ and $D = |\vec{u}||\vec{w}|\cos\theta$

Thus, $\dfrac{\cancel{|\vec{u}||\vec{w}|}\sin\theta}{\cancel{|\vec{u}||\vec{w}|}\cos\theta} = \dfrac{\sqrt{a^2+b^2+c^2}}{D}$ (+2) $\Rightarrow \tan\theta = \dfrac{\sqrt{a^2+b^2+c^2}}{D}$,

so, $\theta = \arctan\left(\dfrac{\sqrt{a^2+b^2+c^2}}{D}\right)$. +1

(solns)

2. (a) [6pts] Find the parametric equations for the line of intersection of the planes

$$3x - 2y + z = 1 \quad \text{and} \quad 2x + y + 3z = 3.$$

Hint: The planes intersect when $x = y = 0$. +1

↳ The Point corresponding to this is $(0,0,1)$ ~ lies on line!

For direction: It is given by the cross product of the normals.

Here, the normals are $3x - 2y + z = 1 \to \vec{n_1} = \langle 3, -2, 1 \rangle$

$2x + y + 3z = 3 \to \vec{n_2} = \langle 2, 1, 3 \rangle$ +1

So, Line's Direction $\vec{V} = \vec{n_1} \times \vec{n_2} = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

$= \langle \det \begin{bmatrix} -2 & 1 \\ 1 & 3 \end{bmatrix}, -\det \begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}, \det \begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix} \rangle$

$= \langle -6 - 1, -(9-2), 3 + 4 \rangle$

$= \langle -7, -7, 7 \rangle$ +2

The line's Equ: $\boxed{x = -7t, \quad y = -7t, \quad z = 1 + 7t}$ +2

(b) [4pts] Find the equation of the plane containing the point (2,0,1) and is perpendicular to the line given by the symmetric equations $\frac{x}{3} = \frac{y-2}{-1} = \frac{z-3}{4}$.

- Line's Direction is $\langle 3, -1, 4 \rangle$ from ↗ and this must be the plane's normal, $\vec{n}$, because it is $\perp$ to the line. +2
- Given pt $(2,0,1)$

+2

Plane Equ: $\boxed{3(x-2) - y + 4(z-1) = 0}$

↳ $\boxed{\text{Same as } 3x - y + 4z = 10}$