• Keep Eyes on your own paper...

Solutions

Math 2D Quiz 4 Evening, October 20th

Please put name on front & ID on back for redistribution!

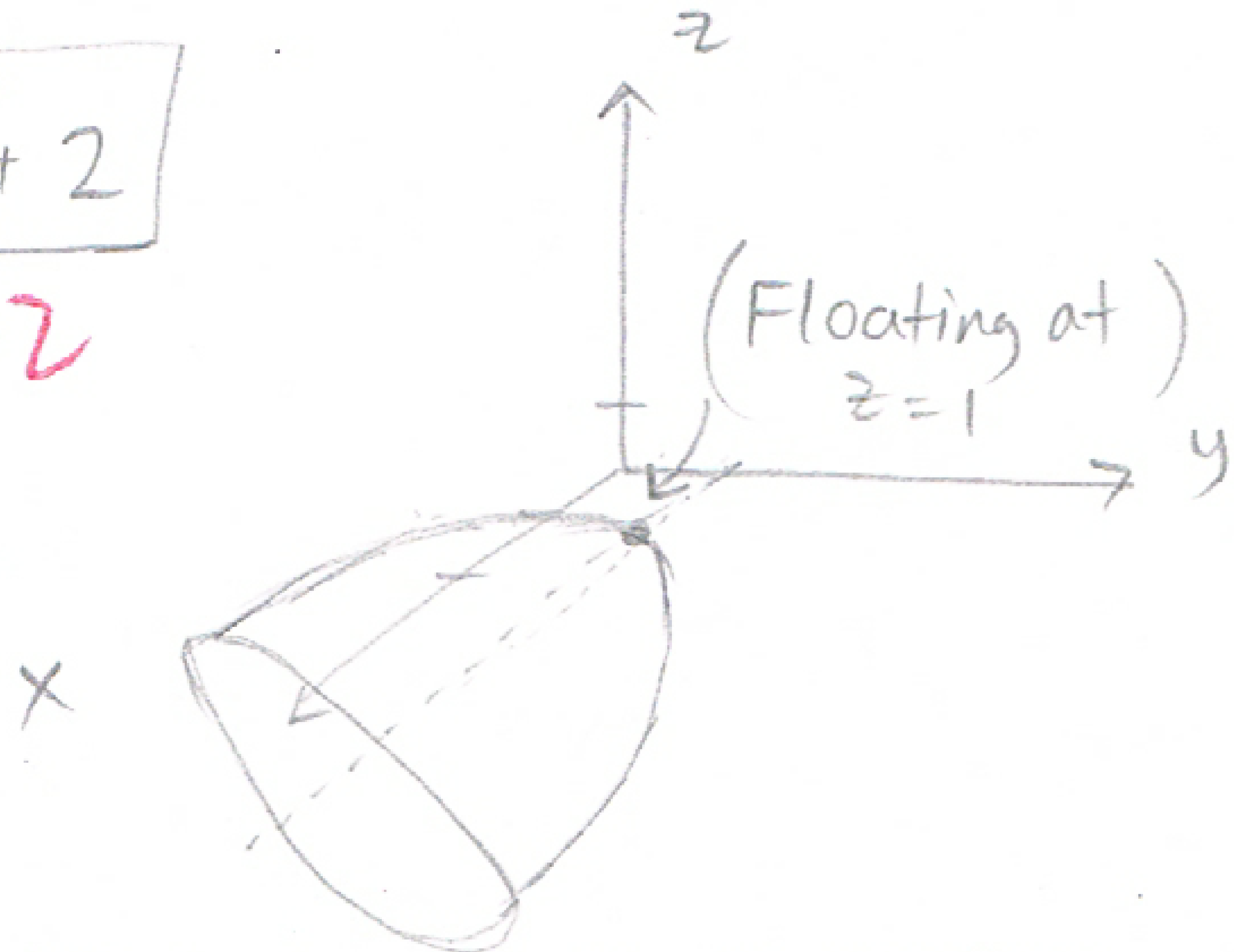
Show all of your work. *There are questions on the back side.*

1. (a) [4pts] Identify and sketch the graph of $x = y^2 + 2z^2 - 2y - 4z + 5$.

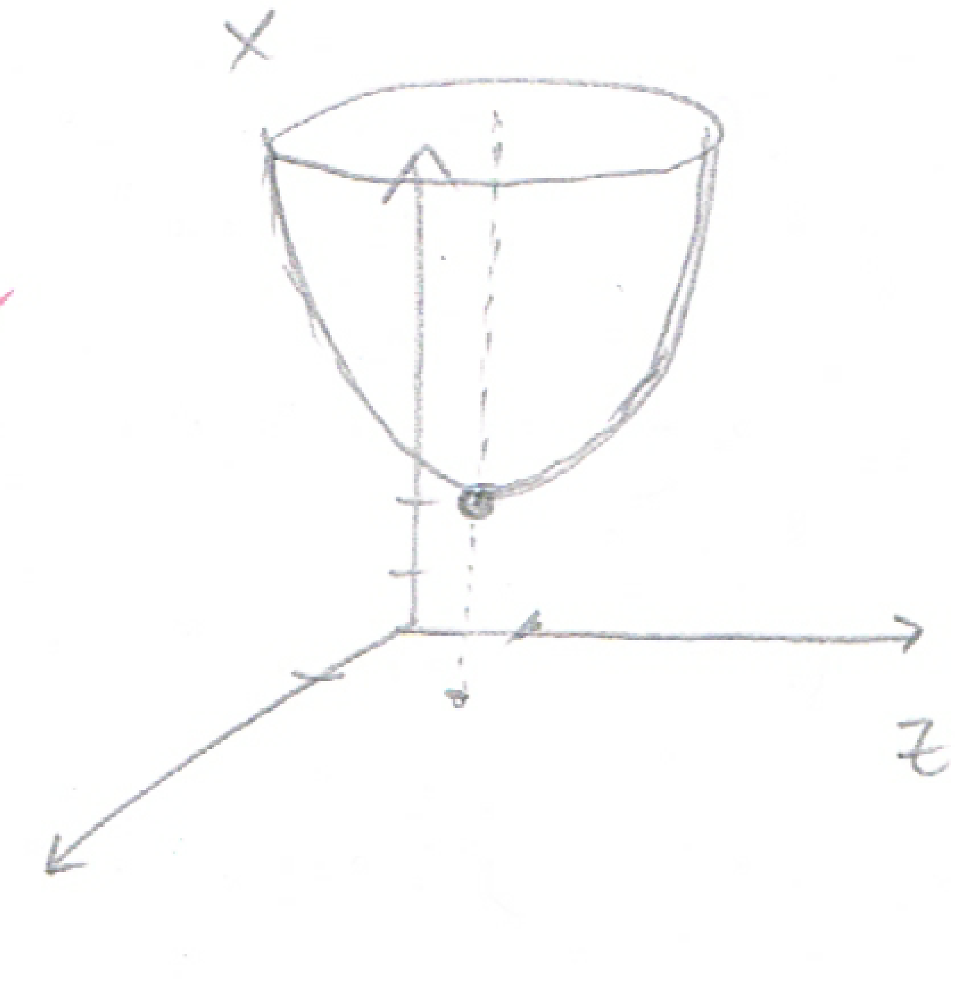
$x = (y-1)^2 + 2(z-1)^2 + 5 - 1 - 2$

ie $\boxed{x = (y-1)^2 + 2(z-1)^2 + 2}$

Elliptic Paraboloid +2



+2 or,

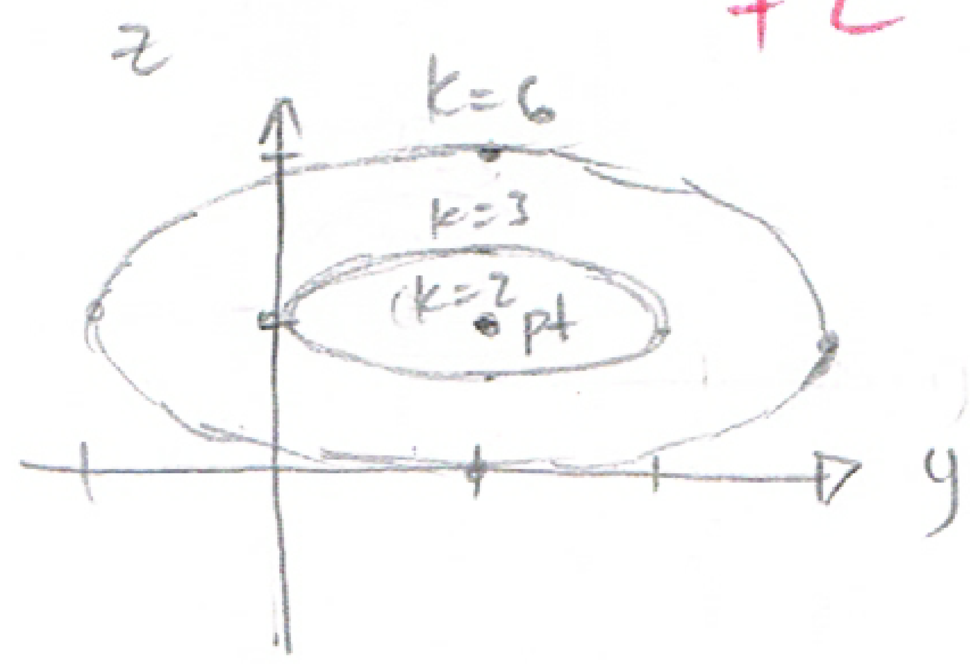


(b) [4pts] Sketch the following traces for the above surface:
(i) yz traces when $x = k$ for $k = 2, 3, 6$. (ii) xz traces when $y = k$ for $k = -1, 0, 1, 2, 3$.

(i) $(y-1)^2 + 2(z-1)^2 = k - 2$

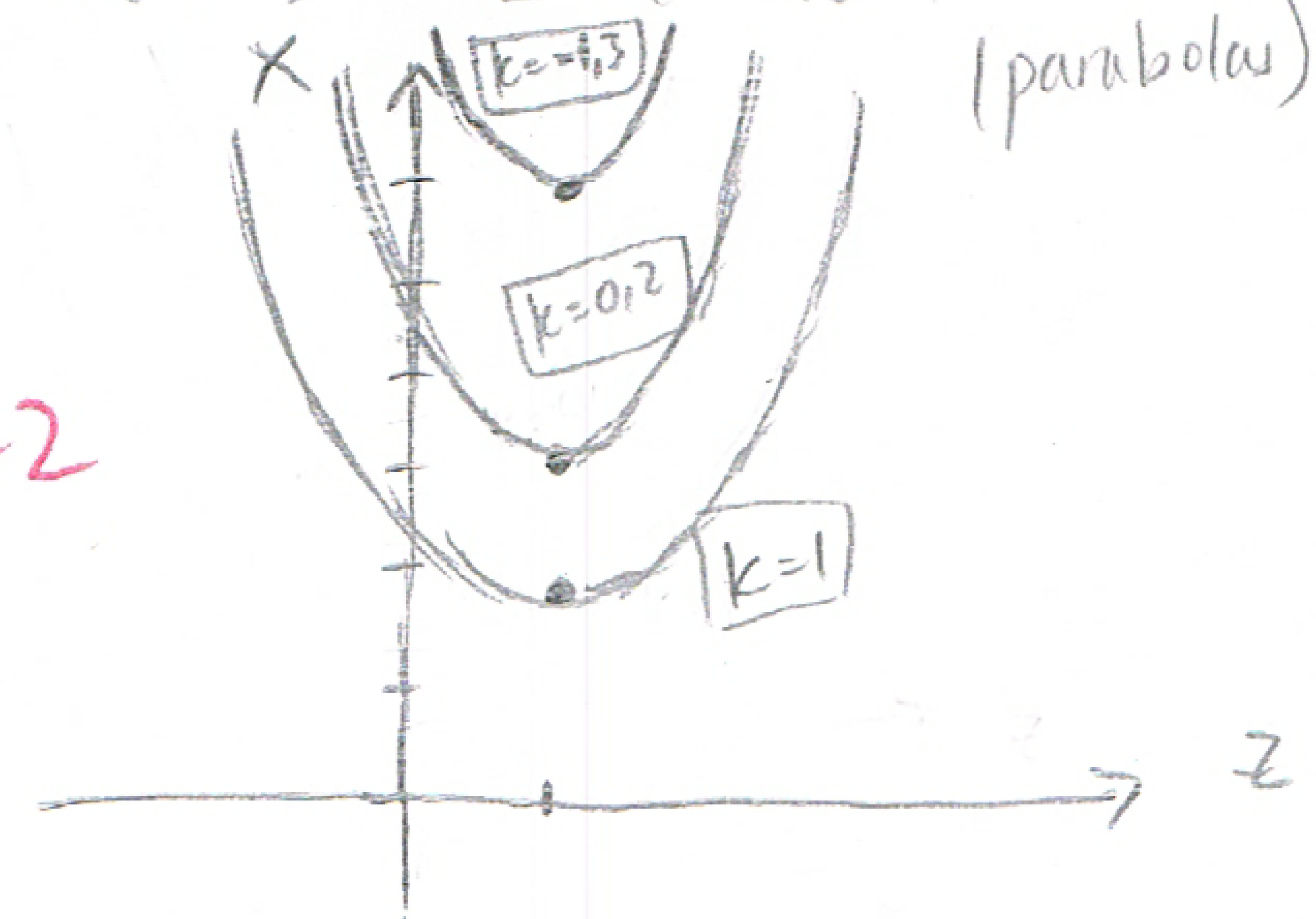
(Ellipses)

+2



(ii) $x = 2(z-1)^2 + 2 + (k-1)^2$ (parabolas)

+2



2. Consider the vector function $\vec{r}(t) = (\sin t)\hat{\imath} + (t^{3/2})\hat{\jmath} + (\cos t)\hat{k}$.
(a) [3pts] Compute the integral $\int_0^\pi \vec{r}(t)$.

+2 as long as answer is a vector

$$= \left. (-)\cos t\,\hat{\imath} + \frac{2t^{5/2}}{5}\hat{\jmath} + \sin t\,\hat{k} \right|_0^\pi$$

$$= 2\hat{\imath} + \frac{2\pi^{5/2}}{5}\hat{\jmath} + 0\hat{k}$$

$$= \boxed{2\hat{\imath} + \frac{2\pi^{5/2}}{5}\hat{\jmath}}$$ +1

(solns)

(b) [3pts] Compute $\vec{r}\,'(t)$. Recall that the curve is $\vec{r}(t) = (\sin t)\hat{\imath} + (t^{3/2})\hat{\jmath} + (\cos t)\hat{k}$.

$$\vec{r}\,'(t) = \cos t\ \hat{\imath} + \frac{3}{2}\sqrt{t}\ \hat{\jmath} - \sin t\ \hat{k}$$

+1 +1 +1

(c) [3pts] Find the equation of the tangent line at $t = \pi$ (here, $\vec{r}(\pi) = \pi^{3/2}\ \hat{\jmath} - \hat{k}$).

At $t = \pi$, $\vec{r}\,'(\pi) = -\hat{\imath} + \frac{3}{2}\sqrt{\pi} - 0\hat{k}$ +1

so Line is $\vec{L}(t) = \langle 0, \pi^{3/2}, -1 \rangle + t\langle -1, \frac{3\sqrt{\pi}}{2}, 0 \rangle$

or, $x = -t$, $y = \pi^{3/2} + \frac{3\sqrt{\pi}}{2}t$, $z = -1$ +2

(d) [3pts] Find the length of the curve from $0 \le t \le 4\pi$. Hint: Reuse part (b).

$$L = \int_0^{4\pi} \sqrt{\cos^2 t + \frac{9}{4}t + \sin^2 t}\ dt$$

$$= \int_0^{4\pi} \sqrt{1 + \frac{9}{4}t}\ dt \quad +1$$

let $u = 1 + \frac{9t}{4}$, $du = \frac{9}{4}dt \to dt = \frac{4du}{9}$,

$$= \int_1^{1+9\pi} \sqrt{u} \cdot \frac{4}{9}du = \frac{4}{9} \cdot \frac{2}{3} u^{3/2} \Big|_1^{1+9\pi} \quad +1$$

$$= \boxed{\frac{8}{27}\left[(1+9\pi)^{3/2} - 1\right]} \quad +1$$

(e) [1pt] **Optional** Sketch the curve $\vec{r}(t) = (\sin t)\hat{\imath} + (t^{3/2})\hat{\jmath} + (\cos t)\hat{k}$ with $t \ge 0$.
If you are missing any credit on this quiz, I will add one point back for a good graph! +1