Solns

Math 2D Quiz 6 Evening, November 10th

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Show all of your work. *There is a question on the back side.*

1. [8pts] Find the equation of the tangent plane to the surface $x^2 + 2y + z = e^{2x+y}$ at $(0, 0, 1)$. What is the equation of the normal line to the surface at this point?

14.6 Perspective:

$F(x,y,z) = x^2+2y+z-e^{2x+y}$, +2

$\nabla F = \langle 2x - 2e^{2x+y}, \ 2 - e^{2x+y}, \ 1 \rangle$

so $\nabla F(0,0,1) = \langle -2, 1, 1 \rangle$. +2

Thus, the Tan Plane is +2

$\boxed{-2(x-0)+(y-0)+(z-1)=0}$

and the Normal line is

$\boxed{\frac{x}{-2} = y = z-1}$ +2

14.4 Perspective:

$z = e^{2x+y} - x^2 - 2y$ then, +1

so $f(x,y) =$ the RHS. +2

$\frac{\partial z}{\partial x} = f_x = 2e^{2x+y} - 2x$; $\frac{\partial z}{\partial y} = f_y = e^{2x+y} - 2$.

At $(0,0)$, $\boxed{f_x(0,0) = 2, \ f_y(0,0) = -1}$.

$\therefore$ The linearization of z is +2

$z - 1 = 2(x-0) - 1(y-0)$, +2

Tan Plane: $\boxed{2x - y - (z-1) = 0}$

And the Normal Line: $\boxed{\frac{x}{2} = -y = 1-z}$ +1

2. [4pts] Find the directional derivative of $g(x, y, z) = \ln(x + y^2 - 3z^3)$ at $(3, 3, -1)$ in the direction of $\vec{u} = \langle 2, 6, 3 \rangle$. In other words, find $D_{\vec{u}}\, g(3, 3, -1)$.

1st $\nabla g(x,y,z) = \left\langle \frac{1}{x+y^2-3z^3}, \frac{2y}{x+y^2-3z^3}, \frac{-9z^2}{x+y^2-3z^3} \right\rangle$.

$= \frac{1}{x+y^2-3z^3} \langle 1, 2y, -9z^2 \rangle$. +1

At $(3,3,-1)$, $\nabla g(3,3,-1) = \frac{1}{15}\langle 1, 6, -9 \rangle$ +1

$2+36-27 = 11$

$\therefore D_{\vec{u}}\, g(3,3,-1) = \nabla g(3,3,-1) \cdot \frac{\vec{u}}{|\vec{u}|}$ +1

Don't forget to Renormalize! +1

$= \frac{1}{15}\langle 1, 6, -9 \rangle \cdot \frac{1}{\sqrt{49}} \langle 2, 6, 3 \rangle = \boxed{\frac{1}{105}(11)}$

(solns)

3. (a) [4pts] Find $\frac{\partial x}{\partial z}$ for the equation $yz + x\ln y = z^2 + e^{y^2} + \ln^2(y)$.

1st $\quad F(x,y,z) = z^2 + e^{y^2} + \ln^2(y) - yz - x\ln y$. +1

$$\frac{\partial x}{\partial z} = \ominus \frac{F_z}{F_x} \quad \text{by Implicit Fn. Thm}$$

$$F_z = 2z - y \qquad \Rightarrow \qquad \boxed{\frac{\partial x}{\partial z} = +\frac{2z-y}{+\ln y}} \quad +1$$

$$F_x = -\ln y \quad +2$$

☆ Note: This only makes sense when $F_x \neq 0$ (and when F_x is defined)

(b) [4pts] Let $g(x,y) = (x^2+y^2)^N$ where $x = r\cos\theta$, $y = r\sin\theta$ and N is an integer $N \geq 1$. Compute using the Chain Rule the derivatives $\frac{\partial g}{\partial r}$ and $\frac{\partial g}{\partial \theta}$. (You must use Chain rule).

[Note: $g \sim r^{2N}$.]

- $$\frac{\partial g}{\partial r} = \frac{\partial g}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial g}{\partial y}\cdot\frac{\partial y}{\partial r}$$

$$= \boxed{2Nx(x^2+y^2)^{N-1}\cdot\cos\theta + 2Ny(x^2+y^2)^{N-1}\cdot\sin\theta} \quad +2$$

[Note: This is like $(2r\cos^2\theta + 2r\sin^2\theta)Nr^{2(N-1)} = 2Nr^{2N-1}$, makes sense] ✓

- $$\frac{\partial g}{\partial \theta} = \frac{\partial g}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial g}{\partial y}\cdot\frac{\partial y}{\partial \theta} \quad +2$$

$$= \boxed{2Nx(x^2+y^2)^{N-1}\cdot(-r\sin\theta) + 2Ny(x^2+y^2)^{N-1}\cdot(r\cos\theta)}$$

[Note: This is $(r^2)^{N-1}\cdot N\left[-2r^2\sin\theta\cos\theta + 2r^2\sin\theta\cos\theta\right] = 0$, Also sensible] ✓