Solns

Math 2D Quiz 6 Morning, November 10th

Please put name on front & ID on back for redistribution!

Show all of your work. *There is a question on the back side.*

1. [8pts] Find the equation of the tangent plane to the surface $xyz = 6$ at the point $(1, 2, 3)$. What is the equation of the normal line to the surface at the point?

14.6 Perspective (Best)

$\nabla F(x,y,z) = \langle yz, xz, xy \rangle$ +2

$\rightsquigarrow \boxed{\nabla F(1,2,3) = \langle 6,3,2 \rangle}$ +2

Thus, the tan. plane eqn:

$\boxed{6(x-1)+3(y-2)+2(z-3)=0}$ +2

And the normal line:

$\boxed{\frac{x-1}{6} = \frac{y-2}{3} = \frac{z-3}{2}}$ +2

14.4 Perspective (for practice)

Write $z = 6/xy$, so $f(x,y) = \frac{6}{xy}$. +1

Then $f_x = -\frac{6}{x^2y}$ +1, $f_y = -\frac{6}{xy^2}$ +1

At $(1,2)$, $\boxed{f_x(1,2) = -3, \quad f_y(1,2) = -\frac{3}{2}}$ +2

$\therefore$ The Linearization of z is

$z - 3 = -3(x-1) - \frac{3}{2}(y-2)$

$\Rightarrow$ Tan plane is +2

$\boxed{3(x-1) + \frac{3}{2}(y-2) + (z-3) = 0}$

And the Normal line is +1

$\boxed{\frac{x-1}{3} = \frac{2(y-2)}{3} = z-3}$

$,z$ too.

2. [4pts] Let $f(x, y) = xe^{xyz}$. Find the maximum rate of change of f at $(2, 1, 0)$ and find the unit direction vector in which it occurs.

$\nabla f(x,y,z) = \langle e^{xyz} + xyze^{xyz}, x^2ze^{xyz}, x^2ye^{xyz} \rangle$ +1

At $(2,1,0)$ $\rightsquigarrow$ $\boxed{\nabla f(2,1,0) = \langle 1, 0, 4 \rangle}$ +1

Thus, the max rate of change is $\boxed{|\nabla f(2,1,0)| = \sqrt{17}}$ +1

And the direction it occurs is the direction of the gradient, $(\pm)$,

$\boxed{\hat{d} = \frac{\nabla f}{|\nabla f|} = \pm\frac{1}{\sqrt{17}}\langle 1, 0, 4 \rangle}$ +1

(solns)

3. (a) [4pts] Find $\frac{\partial x}{\partial y}$ for the equation $y\cos x + e^{z^2+z} = x^2 + y^2 + \arctan(z)$. +1

We know $\frac{\partial x}{\partial y} = \frac{-F_y}{F_x}$ when $F(x,y,z) = x^2 + y^2 + \tan^{-1} z - e^{z^2+z} - y\cos x$

$F_y = 2y - \cos x$ +1

$F_x = 2x + y\sin x$ +1

$\rightsquigarrow$ $\boxed{\frac{\partial x}{\partial y} = \ominus \frac{2y - \cos x}{2x + y\sin x}}$ +1

★ But will dock heavy.

Note: This is only valid when $\boxed{F_x \neq 0}$

(b) [4pts] Let $f(x, y) = (x - y)^5 + (x + y)^7$ where $x = u + w$ and $y = u - w$. Compute, using the Chain Rule, the derivatives $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial w}$.

[Note: $f \sim (2w)^5 + (2u)^7$]

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$= \left[\cancel{5(x-y)^4} + 7(x+y)^6\right]\cdot 1 + \left[\cancel{-5(x-y)^4} + 7(x+y)^6\right]\cdot 1$$

$$= \boxed{14(x+y)^6}$$ +2

[Note: This is $14\cdot(2u)^6$] ✓ (Makes sense)

$$\frac{\partial f}{\partial w} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial w} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial w}$$

$$= \left[5(x-y)^4 + \cancel{7(x+y)^6}\right]\cdot 1 + \left[-5(x-y)^4 + \cancel{7(x+y)^6}\right]\cdot(-1)$$

$$= \boxed{10(x-y)^4}$$ +2

[Note: This is $10\cdot(2w)^4$] ✓ (Also sensible)

(-1) Not simplifying.