$\boxed{\text{solutions}}$

Math 2D Quiz 7 Morning, November 17th

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Show all of your work. *There is a question on the back side.*

1. Consider the function $f(x, y) = x + y - xy$ on the domain $\mathcal{T}$ which is the closed triangular region with vertices (0,0), (0,4), and (2,0).

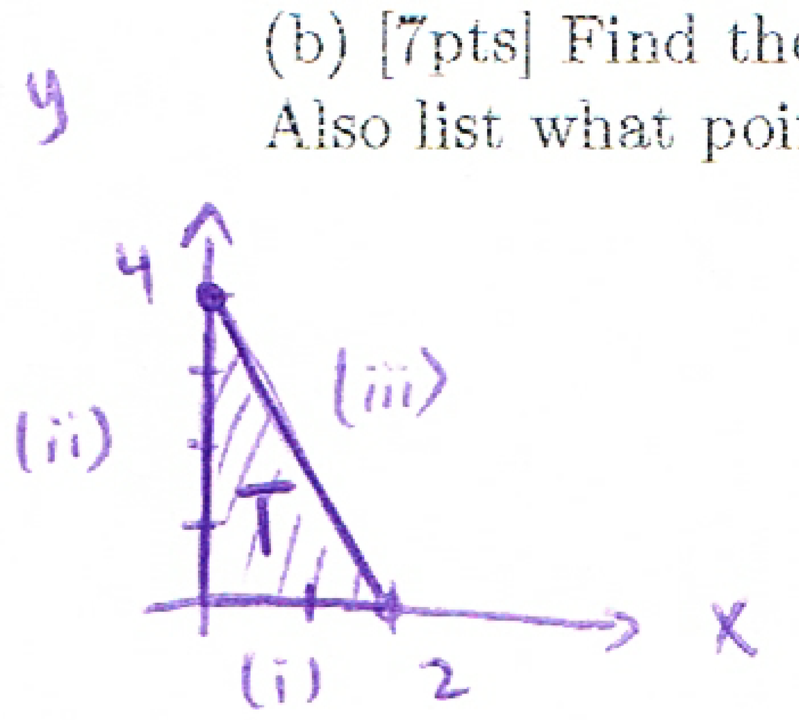
(a) [3pts] Find and classify all critical points of f that are in $\mathcal{T}$. Evaluate f at these points.

$\nabla f(x,y) = \langle 1-y, 1-x \rangle$ +1 is $(0,0)$ at $\boxed{x=1, y=1}$ +1

$D = \det \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx} \cdot f_{yy} - (f_{xy})^2$ +1

$= 0 \cdot 0 - (-1)^2 = \underline{-1 < 0} \Rightarrow \boxed{(1,1) \text{ is a saddle pt}}$

(b) [7pts] Find the absolute max and min of $f(x, y)$ on the domain $\mathcal{T}$. It may help to draw $\mathcal{T}$. Also list what points the absolute max and min occur at.



• Crit. Pt(s) in (a) ✓ so Need Boundary max/min & corner pts.

$\boxed{(i)}$ $y=0$ $x \sim$ var. $f(x,0) = x$, $\underline{\text{line}}$.

+2 ↳ know largest at $x=2$ (corner pts), smallest at $\underline{x=0}$.

$\boxed{(ii)}$ $x=0$, $y \sim$ var, $f(0,y) = y$, line again → Largest at $y=4$. Smallest at $\underline{y=0}$.

+2

$\boxed{(iii)}$ Is line $\underline{y = 4-2x}$ ⇝ $f(x, 4-2x) = x + 4 - 2x - (4x - 2x^2)$.

+2

$= 2x^2 - 5x + 4$.

$\frac{df}{dx} = 4x - 5$ ⇝ $x = \frac{5}{4}$ critical $\Rightarrow$ $\left(\frac{5}{4}, \frac{3}{2}\right)$ critical.

$\left(y = 4 - \frac{5}{2} = \frac{3}{2}\right)$

From (a)

+1 $\underline{\text{Compare}}$: • $f(1,1) = 1$

From (ii) { • $\boxed{f(0,4) = 4}$ max

From (i) { • $f(2,0) = 2$ • $\boxed{f(0,0) = 0}$ min

From (iii) { • $f\left(\frac{5}{4}, \frac{3}{2}\right) = \frac{50}{16} - \frac{25}{4} + 4$

$= -\frac{25}{8} + 4 = \frac{7}{8}$

(solns)

$g \qquad h$

2. [10pts total] Find the point(s) on the curve of intersection of $x - y = 1$ and $y^2 - z^2 = 1$ that are closest in distance^2 to the origin. Also find this distance^2. Follow this outline:

(a) [2pts] What is the system of equations we have to solve for Lagrange Multipliers?
Recall: The distance^2 to the origin (0,0,0) would be the function $f(x, y, z) = x^2 + y^2 + z^2$.

$\nabla f = \lambda \nabla g + \mu \nabla h$

$x - y = 1$

$y^2 - z^2 = 1$

$$\leadsto \begin{cases} (i) & 2x = \lambda \\ (ii) & 2y = -\lambda + 2\mu y \\ (iii) & 2z = -2\mu z \\ (iv) & x - y = 1 \\ (v) & y^2 - z^2 = 1 \end{cases} \qquad +2$$

(b) [4pts] Trivial Cases. How can any of the above equations read as $0 = 0$? Hint: Look at LHS. What are the corresponding points we get?

↳ This only applies to (i) → (iii) !!
B/c (iv) & (v) are constraints, set values already

If (i) is $0=0$: +1

$x = 0$ then
(iv) $\Rightarrow y = -1$
(v) $\Rightarrow z = 0$

$\boxed{(0, -1, 0)}$
kept.

If (ii) is $0=0$: +1

$y = 0$ then
(iv) $\Rightarrow x = 1$
(v) $\Rightarrow z^2 = -1$??

$\boxed{\text{No Soln!}}$

If (iii) is $0=0$:

$z = 0$ then,
(v) $\Rightarrow y = \pm 1$
(iv) $\Rightarrow x = 2, 0$ respectively. +1

$\boxed{(2,1,0), \quad (0,-1,0)}$
2 pts kept. duplicated

(+1 if all ok).

(c) [4pts] What solutions do we get to the system in (a) assuming the nonzero cases? Are there any? What is the minimal distance^2, i.e. $f(x, y, z)$, among the points in (b) and (c)?

When none are $0=0$: In (iii), $\Rightarrow \mu = -1$ then.

Then (ii) $\to 2y = -\lambda - 2y \Rightarrow 4y = -\lambda$ and $2x = \lambda$ from (i).

$\therefore 2x = -4y$, $\boxed{x = -2y}$. Plug into (iv) $\Rightarrow -3y = 1$. +3

we get $y = -1/3 \longrightarrow$ (v) is $\frac{1}{9} - z^2 = 1$ has no solution!

$\therefore$ our ~~all~~ pts in (b) are the only pts.

$\boxed{f(0,-1,0) = 1}$ min +1 $\boxed{f(2,1,0) = 5}$ max