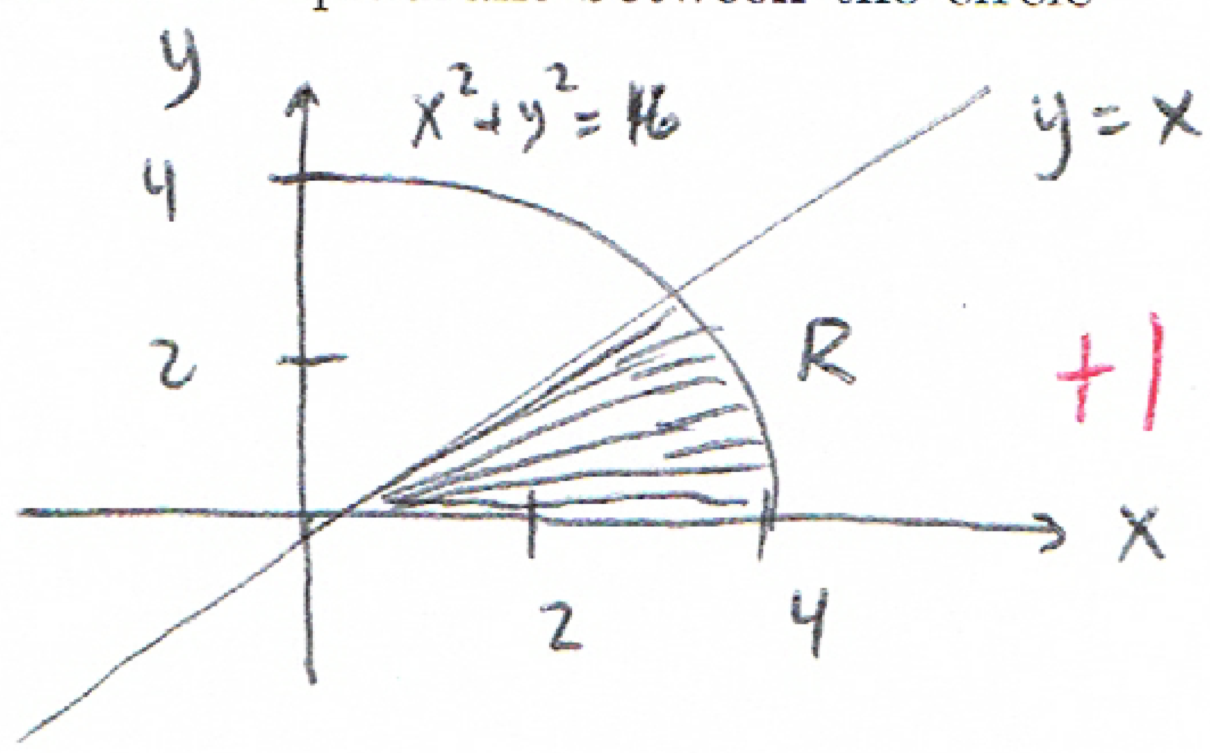
Solutions

Math 2D Evening Last Quiz - December 1st

Please put name on front & ID on back for redistribution!

Show all of your work. *There is a question on the back side.*

1. [10pts] Compute $\iint_R (2x - y)\,dA$ where R is the region in the 1st quadrant between the circle of radius 4 and the lines $y = x$ and $y = 0$. " x-axis.
Please also sketch the region, R.



+1

- We see θ is from 0 to $\pi/4$

(The line $y=x$ creates an angle of $\tan^{-1}(\text{slope}) = \tan^{-1}(1) = \pi/4$)

- r goes from 0 to 4.

ie $r^2(2\cos\theta - \sin\theta)$

Integral $\overset{\text{Polar}}{=} \int_{\theta=0}^{\pi/4} \int_{r=0}^{4} (2r\cos\theta - r\sin\theta)\cdot r\,dr\,d\theta$ +2 +2 +2

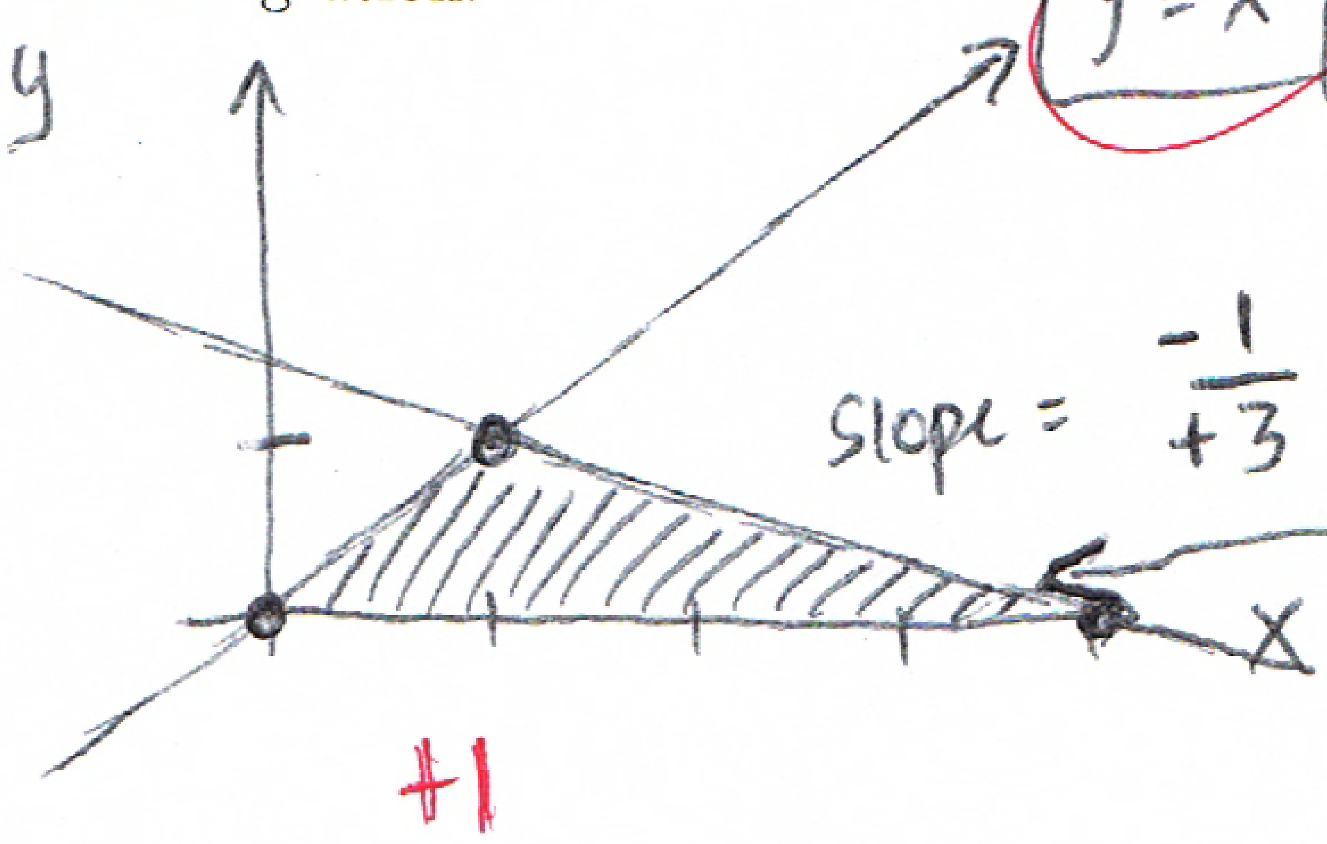
$= \int_0^{\pi/4} \frac{r^3}{3}\Big|_0^4 \cdot (2\cos\theta - \sin\theta)\,d\theta$ +1   * $\frac{r^3}{3}\Big|_0^4 = \frac{64}{3}$.

$= \frac{64}{3}\cdot\left[2\sin\theta + \cos\theta\right]\Big|_{\theta=0}^{\pi/4} = \frac{64}{3}\left[\left(\frac{2+1}{\sqrt{2}}\right) - (0+1)\right]$ +1

$= \boxed{\frac{64}{3}\left(\frac{3}{\sqrt{2}} - 1\right)}$ +1 (same as: $32\sqrt{2} - \frac{64}{3}$)

2. We will find the volume between the planes $z - y = 1$ and $z = -1$ within the triangular xy region with vertices at $(0,0)$, $(1,1)$, $(4,0)$.
(a) [1pt] Sketch the domain of integration.

$\boxed{y=x}$



+1

slope $= \frac{-1}{+3}$ ~>

Use pt $(4,0)$ with pt-slope eqn,

$y = -\frac{1}{3}(x-4)$

ie. $x = 4 - 3y$

$\boxed{\text{(solns)}}$

(b) [6pts] Set up the integral that finds the volume in *both* orders, as $dxdy$ and as $dydx$. Be sure to have the correct integral bounds and the correct function inside the integrals.

Here, $z_{top} = 1+y$ and $z_{bot} = -1$, so,

$$\boxed{\text{I}} \quad \int_{x=0}^{1} \int_{y=0}^{x} (y+2)\,dy\,dx + \int_{x=1}^{4} \int_{y=0}^{\frac{4-x}{3}} (y+2)\,dy\,dx \qquad +4$$

$$\boxed{\text{II}} \quad \int_{y=0}^{1} \int_{x=y}^{4-3y} (y+2)\,dx\,dy. \qquad +2$$

(Easier!) (c) [3pts] Compute the volume using one of the integrals in part (b). $+1$

$$\boxed{\text{II}}: \; = \int_{y=0}^{1} (y+2)\cdot x \Big|_{x=y}^{4-3y} dy = \int_0^1 \Big((4y+8-3y^2-6y) - (y^2+2y) \Big) dy$$

$$= \int_{y=0}^{1} \left(-4y^2 - 4y + 8\right) dy = -\frac{4y^3}{3} - 2y^2 + 8y \Big|_{y=0}^{1} \qquad +1$$

$$= -\frac{4}{3} - 2 + 8 = 6 - \frac{4}{3} = 4\tfrac{2}{3} = \boxed{\frac{14}{3}} \qquad +1$$

$\boxed{\text{OR}}$

$$\boxed{\text{I}} = \int_{x=0}^{1} \frac{y^2}{2} + 2y \Big|_{y=0}^{x} dx + \int_{x=1}^{4} \frac{y^2}{2} + 2y \Big|_{y=0}^{\frac{4-x}{3}} dx$$

(Hard!!)

$$= \int_{x=0}^{1} \left(\frac{x^2}{2} + 2x\right) dx + \int_{x=1}^{4} \left(\frac{16-8x+x^2}{18} + \frac{8-2x}{3}\right) dx \qquad +1 \qquad \left(\frac{23-9}{3} = \right.$$

$$= \frac{x^3}{6} + x^2 \Big|_0^1 + \left[\frac{8}{9}x + \frac{8}{3}x - \frac{4x^2}{18} - \frac{x^2}{3} + \frac{x^3}{54}\right]\Big|_{x=1}^{4} \qquad +1 \qquad +1$$

$$= \frac{7}{6} + \Big(\underbrace{\frac{128}{9} - \frac{160}{18}}_{+96/18 \,=\, +\frac{16}{3}} + \underbrace{\frac{64}{54}}_{+\frac{7}{6}}\Big) - \Big(\underbrace{\frac{32}{9} - \frac{10}{18}}_{54/18 \,=\, 3} + \frac{1}{54}\Big) = \frac{7+16}{3} - 3 = \boxed{\frac{14}{3}}$$

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