Solns

Math 2D Morning Last Quiz - December 1st

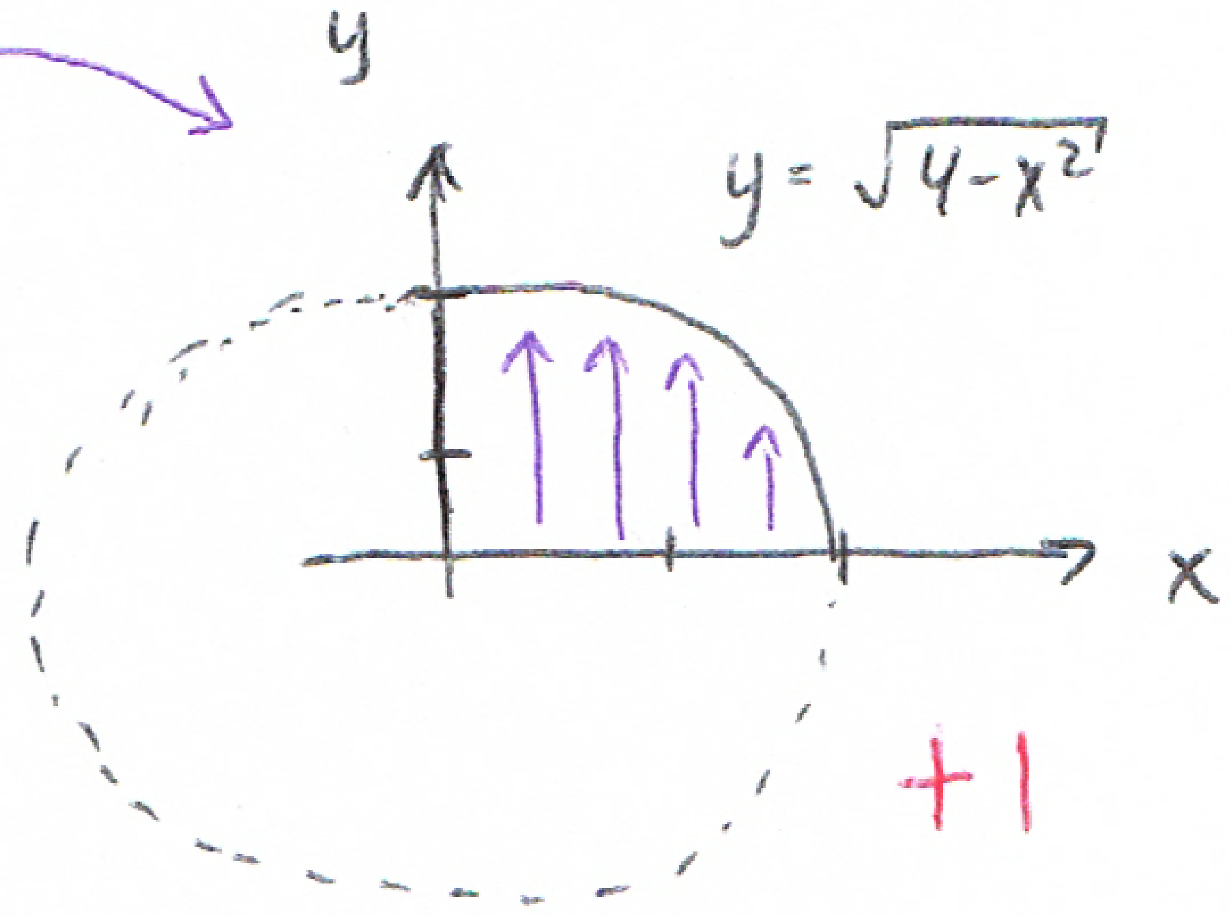
Please put name on front & ID on back for redistribution!

Show all of your work. *There is a question on the back side.*

1. [10pts] Compute the integral of

$$\boxed{I} = \int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2}\, dy dx.$$

Please sketch the domain of integration, too.



+1

- We can cover this 1/4 of a circle by having $0 \le \theta \le \frac{\pi}{2}$ and $0 \le r \le 2$.

+2 θ-bds
+2 r-bds

so, $I = \int_{\theta=0}^{\pi/2} \int_{r=0}^{2} e^{-r^2} \cdot r\, dr d\theta$ +1

☆ let $u = r^2$
$du = 2r dr$
$(dr = \frac{du}{2r})$ +1

$= \int_{\theta=0}^{\pi/2} \int_{u=0}^{4} \frac{e^{-u}}{2}\, du\, d\theta = \frac{1}{2} \int_{\theta=0}^{\pi/2} -e^{-u} \Big|_{u=0}^{4} d\theta$ +2

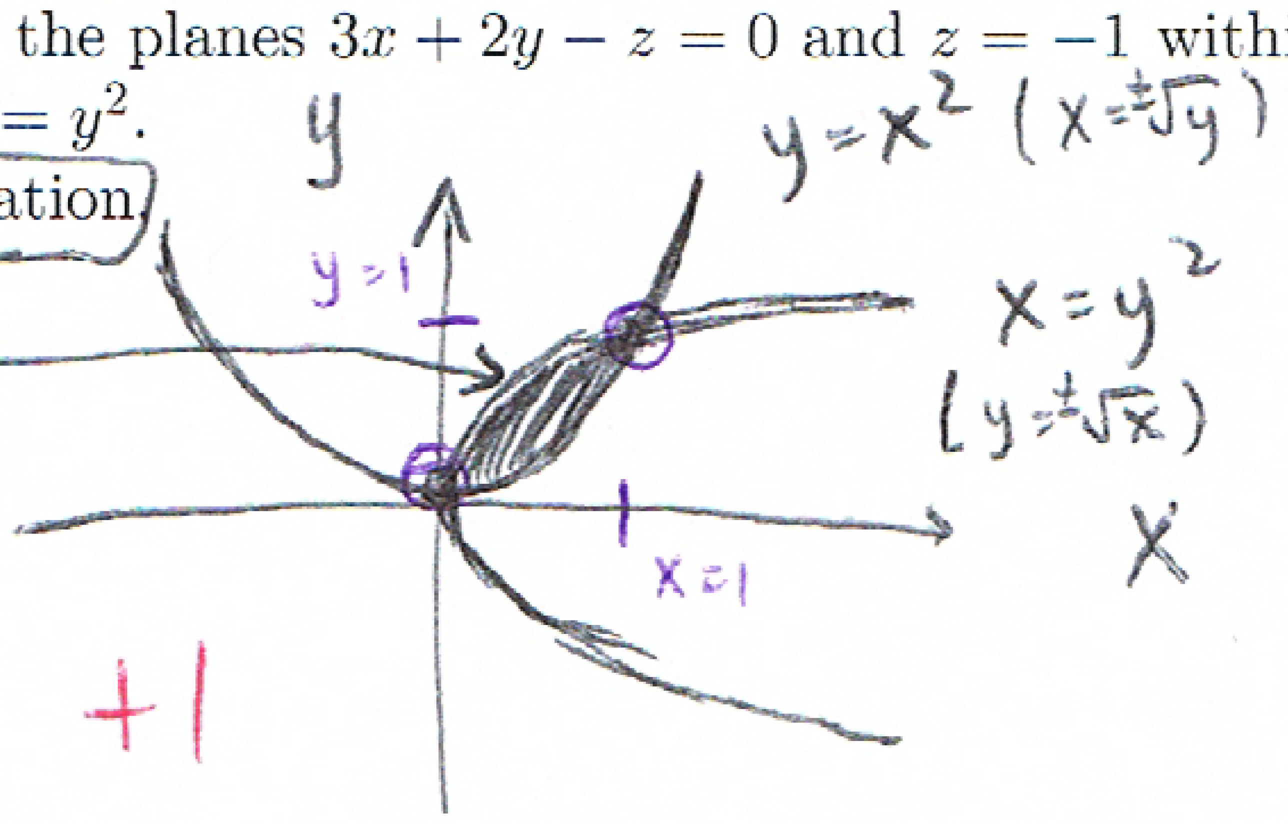
θ-independent

$\overset{\theta\text{-independent}}{=} \frac{1}{2} \cdot \frac{\pi}{2} \cdot (-e^{-u}) \Big|_{u=0}^{4} = \frac{\pi}{4}\left(-e^{-4} + e^{0}\right)$

$= \boxed{\frac{\pi}{4}\left(1 - e^{-4}\right)}$ +1

2. We will find the volume between the planes $3x + 2y - z = 0$ and $z = -1$ within the xy region bounded by the curves $y = x^2$ and $x = y^2$.

(a) [1pt] Sketch the domain of integration.



+1

- In particular: $\sqrt{x} = x^2$ when $\boxed{x = 0, 1}$ ⇝ $\boxed{y = 0, 1}$ on intersections

(solus)

Also, $z_{top} = 3x+2y$, $z_{bot} = -1$

(b) [6pts] Set up the integral that finds the volume in *both* orders, as $dxdy$ and as $dydx$. Be sure to have the correct integral bounds and the correct function inside the integrals.

$\boxed{\text{I}}$: $\displaystyle\int_{x=0}^{1}\int_{y=x^2}^{\sqrt{x}} (3x+2y+1)\,dy\,dx$ +3

$\boxed{\text{II}}$: $\displaystyle\int_{y=0}^{1}\int_{x=y^2}^{\sqrt{y}} (3x+2y+1)\,dx\,dy.$ +3

(c) [3pts] Compute the volume using one of the integrals in part (b).

$\boxed{\text{I}}$:

$$= \int_{x=0}^{1} 3xy+y^2+y \Big|_{y=x^2}^{\sqrt{x}} dx$$

$$= \int_{x=0}^{1} \left((3x^{3/2}+x+x^{1/2}) - (3x^3+x^4+x^2) \right) dx$$ +1

$$= \left[\frac{6x^{5/2}}{5} + \frac{x^2}{2} + \frac{2x^{3/2}}{3} - \frac{3x^4}{4} - \frac{x^5}{5} - \frac{x^3}{3} \right]_{x=0}^{1}$$ +1

$$= \left(\frac{6}{5} + \frac{1}{2} + \frac{2}{3} - \frac{3}{4} - \frac{1}{5} - \frac{1}{3} \right) - (0)$$

$$= \frac{1}{60}\left(72+30+40\underbrace{-45-12-20}_{-77} \right)$$

$$= \frac{70-5}{60} = \frac{65}{60} = \boxed{\frac{13}{12}}$$ +1

$\boxed{\text{II}}$:

$$= \int_{y=0}^{1} \frac{3x^2}{2} + 2yx + x \Big|_{x=y^2}^{\sqrt{y}} dy$$

$$= \int_{y=0}^{1} \left[\left(\frac{3y}{2} + 2y^{3/2} + y^{1/2} \right) - \left(\frac{3y^4}{2} + 2y^3 + y^2 \right) \right] dy$$

$$= \frac{3y^2}{4} + \frac{4y^{5/2}}{5} + \frac{2y^{3/2}}{3} - \frac{3y^5}{10} - \frac{y^4}{2} - \frac{y^3}{3} \Big|_{y=0}^{1}$$

$$= \left(\frac{3}{4} + \frac{4}{5} + \frac{2}{3} - \frac{3}{10} - \frac{1}{2} - \frac{1}{3} \right) - (0)$$

$$= \frac{1}{60}\left(45+\underbrace{48+40}_{88}\underbrace{-18-30-20}_{-68} \right)$$

$$= \frac{1}{60}(45+20) = \frac{65}{60} = \boxed{\frac{13}{12}}$$