Solutions

separates

Math 3D Quiz 2 Morning - February 2nd

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Show all of your work. *There is a question on the back side.*

(other way: $xy' = -y^2 - y$)

1. (a) [8pts] Solve $xy' + y + y^2 = 0$ with $y(1) = 2$. You must use a Bernoulli substitution.

$\boxed{n=2}$

1. Let $v = y^{1-2} = y^{-1}$, $v' = -y^{-2}y'$

2. Divide y^2 $\rightsquigarrow$ $xy'y^{-2} + y^{-1} + 1 = 0$

Can use two ways

Eqn in v $\Rightarrow$ $-xv' + v = -1$; $\boxed{v' - \frac{1}{x}v = \frac{1}{x}, \; v(1) = \frac{1}{2}}$. +3

Integration Factor

$R(x) = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = e^{\ln x^{-1}} = \frac{1}{x}$ +1

$\Rightarrow \frac{d}{dx}\left[\frac{1}{x} \cdot v\right] = \frac{1}{x^2}$, integrate,

$\Rightarrow \frac{1}{x} \cdot v = -\frac{1}{x} + c$; $\boxed{v = cx - 1}$ +1

I.C. $\Rightarrow$ $\frac{1}{2} = c - 1 \Rightarrow \boxed{c = \frac{3}{2}}$, $\boxed{v = \frac{3x}{2} - 1}$. +1

3. Re-Sub: $\boxed{y = \left(\frac{3x}{2} - 1\right)^{-1} = \frac{1}{\frac{3x}{2} - 1}}$ +1

Domain: $\frac{3x}{2} - 1 \neq 0$, $x \neq \frac{2}{3}$. I.C. has $x = 1$ so need $\boxed{x > 2/3}$ +1

Separate:

$v' = \frac{1}{x}(1+v)$;

$\frac{dv}{v+1} = \frac{dx}{x} \Rightarrow \ln|v+1| = \ln|x| + C$ +2

exp $\Rightarrow |v+1| = \tilde{C}|x|$

$v(1) = \frac{1}{2} \Rightarrow \boxed{\frac{3}{2} = \tilde{C}}$ +1

$\Rightarrow$ pick both (+) branch

$v + 1 = \frac{3}{2}x$; $v = \frac{3x}{2} - 1$.

3. Re-Sub: $\boxed{y = \frac{1}{\frac{3x}{2} - 1}}$ +

And needs $\boxed{x > 2/3}$ +1

(b) [2pts] Prove that the functions $\{e^{4t}, te^{4t}\}$ are linearly independent.

Short Way: $\frac{e^{4t}}{te^{4t}} = \frac{1}{t} \neq$ constant $\Rightarrow \boxed{\text{L.I.}}$ ✓

Formal Way: For $c_1 e^{4t} + c_2 t e^{4t} = 0$, need $c_1 + c_2 t = 0$ for all t.

Slick Way: • If c_1 or $c_2 \neq 0$, it's a line eqn, is 0 for at best one t-value.

Standard: If $t = 0 \Rightarrow \boxed{c_1 = 0}$ needed then

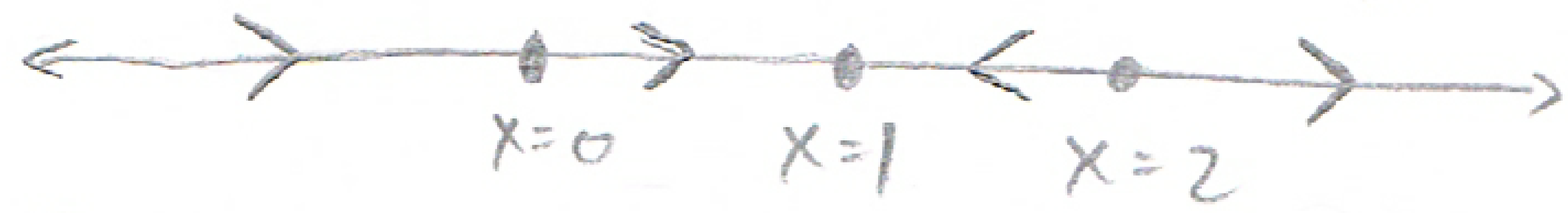
Then if $t = 1 \Rightarrow 0 + c_2 = 0$, $\boxed{c_2 = 0}$. L

$\Rightarrow$ To solve 0-eqn, needs $\boxed{c_1 = c_2 = 0}$, call it $\boxed{f(x)}$

2. [5pts] Let $x' = \overbrace{x^2(x-1)(x-2)}$.
(a) Find and classify all critical points. Sketch the phase diagram, too.
(b) i. If $x(0) = 0.01$ and if ii. $x(0) = 2.1$, what is $\lim_{t\to\infty} x(t)$ for each case, i. and ii.?

$f(-1) > 0$ $\quad f(\frac{1}{2}) > 0$ $\quad f(\frac{3}{2}) < 0$ $\quad f(3) > 0$.

a) zeros are $x^* = 0, 1, 2$.



Stable: $X = 1$ $\qquad$ Unstable: $X = 0, 2$

+1 classified $\qquad$ +2 diagram.

b) $X(0) = 0.01 \Rightarrow$ Goes to $x = 1$ $\quad$ +1

$X(0) = 2.1 \Rightarrow$ Goes to $+\infty$ $\quad$ +1

3. [5pts] Consider the equation $x'' - 8x' + 16x = 0$. (Here, x is a function of t).
(a) Find the general solution to the equation.
(b) Find the exact solution to the equation if $x(0) = 2$, $x'(0) = 0$.

a) Aux Equ $\rightarrow$ $r^2 - 8r + 16 = 0$; +1 $\quad r = \frac{8 \pm \sqrt{64-64}}{2} = 4$

$(r-4)^2 = 0$ $\qquad \boxed{r = 4 \text{ (mult. 2)}}$ +1

$\Rightarrow \boxed{X(t) = C_1 e^{4t} + C_2 t e^{4t}}$ +1 is general solution.

b) $X'(t) = 4C_1 e^{4t} + C_2(e^{4t} + 4te^{4t})$, needed.

$X(0) = 2$: $\quad \boxed{2 = C_1}$ +1 $\qquad X'(0) = 0$: $\quad 0 = 4C_1 + C_2$ $\quad$ +1

$= 8 + C_2$, $\quad \boxed{C_2 = -8}$

$\Rightarrow \boxed{X(t) = 2e^{4t} - 8te^{4t}}$.

$X(0) = 2e^0 - 0 = 2$ ✓

↳ It's good to check w/ I.C: $\quad X'(0) = 8e^0 - 8(e^0 + 0) = 0$ ✓