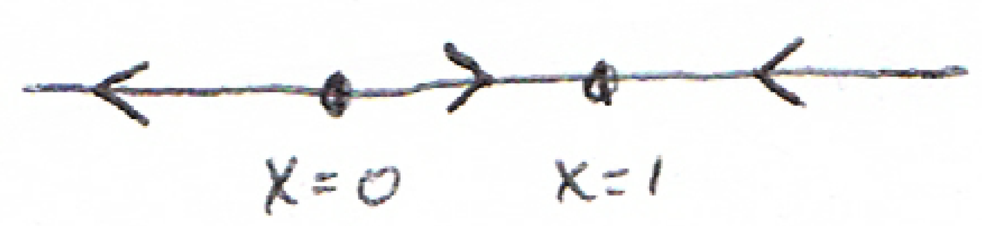
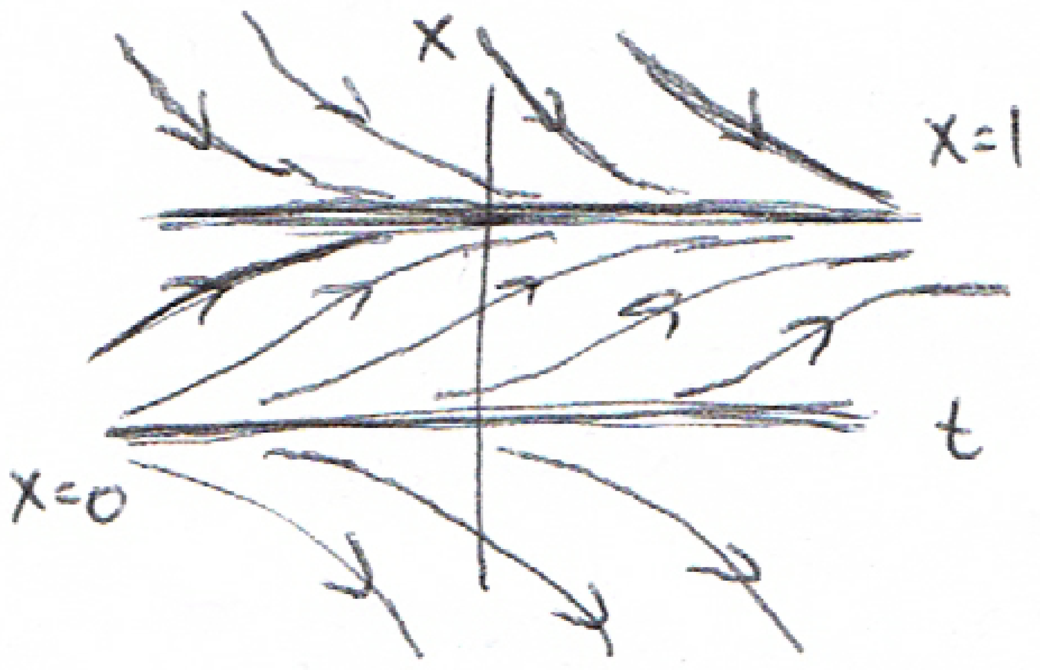
HW 3 Selected Problems

1.6.5) a. 

$\Rightarrow$ $x=0$ unstable crit pt

$x=1$ stable crit pt.

4 pts

b. 

c. If $x(0)=\frac{1}{2}$, as $t\to\infty$, it goes to $\boxed{x=1}$

2.3.6) $y_c = e^{4x}\left(C_1\cos x + C_2\sin x + C_3 x\cos x + C_4 x\sin x\right)$

1 pt

2.3.7) Here, $h=\frac{1}{2}(g-f)$ so Linearly Dependent. [OR follow definition, find nontrivial 0-solution]

2 pts

2.5.7) 1st step, Aux Equ $\rightsquigarrow$ $r^2-2r+1=0$, $(r-1)^2=0$, $r=1$ (mult. 2).

5 pts

So $y_c = C_1e^x + C_2xe^x \rightsquigarrow y_1=e^x$, $y_2=xe^x$.

a) $\begin{cases} u_1'e^x + u_2'xe^x = 0 & \text{(I)} \\ u_1'e^x + u_2'(e^x+xe^x) = e^x & \text{(II)} \end{cases}$ $\Rightarrow$ (II)-(I): $u_2'e^x = e^x$; $\boxed{u_2'=1}$

Plug into (I): $u_1'\cancel{e^x} + x\cancel{e^x} = 0$ $\boxed{u_1' = -x}$

So, $u_1 = -\frac{x^2}{2}$, $u_2 = x$ $\Rightarrow$ $\boxed{y_p = -\frac{x^2}{2}e^x + x^2e^x = \frac{x^2}{2}e^x}$

(Maybe you had $+C_1e^x + C_2xe^x$).

b) Due to multiplicity, guess $y_p = Ax^2e^x$; $y_p' = 2Axe^x + Ax^2e^x$; $y_p'' = 2Ae^x + 4Axe^x + Ax^2e^x$

For $y_p'' - 2y_p' + y_p = e^x$:

e^x: $2A=1 \to \boxed{A=1/2}$

xe^x: $4A-4A=0$ ✓

x^2e^x: $A-2A+A=0$ ✓

$\Rightarrow \boxed{y_p = \frac{x^2}{2}e^x}$

c) I got the same... In general, using variation of parameters may add repeated terms of the complementary solution.