HW 4 selected Problems

2.4.2) a. $2x'' + 8x' + 3x = 0 \rightsquigarrow 2r^2 + r + 3 = 0$; $r = \frac{-1 \pm \sqrt{1-24}}{4} = \frac{-1 \pm i\sqrt{23}}{4}$.

3 pts

General Soln: $x = e^{-t/4}\left(c_1 \cos\frac{\sqrt{23}t}{4} + c_2 \sin\frac{\sqrt{23}t}{4}\right)$.

b. Underdamped c. Needs $\sqrt{c^2-24} = 0 \Rightarrow \boxed{c = 2\sqrt{6}}$, ($c = \sqrt{24}$) only (+) due to physical reasons

4 pts

3.1.3) Since coupled equations $\rightsquigarrow$ Differentiate 2nd, $x_2'' = x_1'$ and plug in for x_1'.

$x_2'' = 3x_1 - x_2 + e^t$ // plug in $x_2' = x_1 \Rightarrow x_2'' = 3x_2' - x_2 + e^t$.

so, solve $x_2'' - 3x_2' + x_2 = e^t$. ($r^2 - 3r + 1 = 0 \rightsquigarrow r = \frac{3 \pm \sqrt{9-4}}{2}$)

$\boxed{x_{2,c} = c_1 e^{at} + c_2 e^{bt}}$ with $\boxed{a = \frac{3+\sqrt{5}}{2}}$, $\boxed{b = \frac{3-\sqrt{5}}{2}}$.

$x_{2,p} = Ae^t$ then, $x_{2,p}' = Ae^t$ and $x_{2,p}'' = Ae^t$ too so plugging in,

$e^t(A - 3A + A) = e^t \Rightarrow \boxed{A = -1}$, $\boxed{x_{2,p} = -e^t}$

So, $\boxed{x_2 = c_1 e^{at} + c_2 e^{bt} - e^t}$ $\xrightarrow{x_2' = x_1}$ $\boxed{x_1 = c_1 a e^{at} + c_2 b e^{bt} - e^t}$

can NOT rename constants! (to say, $\tilde{c}_1, \tilde{c}_2$)

3.1.4) $\begin{bmatrix} y \\ y' \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -\frac{c}{a} & -\frac{b}{a} \end{bmatrix}\begin{bmatrix} y \\ y' \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{f(x)}{a} \end{bmatrix}$ since $\frac{ay''}{a} = \frac{-by' - cy + f(x)}{a}$

2 pts

- Completed all 3.2 problems, with correct approaches.

3 pts (-1) for each missing.