HW 5 Key

3.3.5) Need to show for $c_1\begin{bmatrix} t \\ t^2 \end{bmatrix} + c_2\begin{bmatrix} t^3 \\ t^4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, both c_1, c_2 need to be 0.

2 pts

Row 1: $c_1 t + c_2 t^2 = 0$
Row 2: $c_1 t^2 + c_2 t^4 = 0$ $\Longrightarrow$

Soln 1: Row 1 means $c_1 + c_2 t = 0$ for all t,
$c_1 = -c_2 t$ for all t $\Rightarrow$ $\boxed{c_1 = c_2 = 0}$
is necessary because LHS = constant, for all t.

Soln 2: Since in Row 1, $\{t, t^2\}$ are L.I. functions,
$\left(\frac{t^2}{t} = t \neq \text{constant} \right)$, for $c_1 t + c_2 t^2 = 0$, L.I $\Rightarrow$ $c_1 = c_2 = 0$

Either solution $\Rightarrow$ c_1, c_2 both 0 $\Rightarrow$ $\left\{ \begin{bmatrix} t \\ t^2 \end{bmatrix}, \begin{bmatrix} t^3 \\ t^4 \end{bmatrix} \right\}$ is L.I.

3.7.4) 5 pts

a. $\lambda = 2$ (Alg. Mult. of 3).

- upper-triangular, so e-vals on diagonal.
- $\det \begin{bmatrix} 2-\lambda & 1 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{bmatrix} = (2-\lambda) \cdot \det \begin{bmatrix} 2-\lambda & 1 \\ 0 & 2-\lambda \end{bmatrix}$
$= (2-\lambda)^3$.

b. $(P-2I)\vec{x} = \vec{0}$: $\left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\Rightarrow$ x_1 = free, $x_2 = 0$, x_3 = free, $\boxed{\vec{x} = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$

We have 2 eigenvectors $\Rightarrow$ GM = 2, only once defective. (AM − GM = 1)

1st way

c. Need one more generalized eigenvector. Note $(P-2I)\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ has no solution

So, we get it by solving $(P-2I)\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\left[\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\Rightarrow$ x_1 = free, $x_2 = 1$, x_3 = free

So our generalized e-vector is $\boxed{\vec{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$

Since it came from the eigenvector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, our general soln is

$$\boxed{\vec{x}_{gen} = c_1 e^{2t} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_3 e^{2t} \left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right)}$$

As a matrix, $\boxed{\vec{x}_{gen} = \begin{bmatrix} 0 & e^{2t} & te^{2t} \\ 0 & 0 & e^{2t} \\ e^{2t} & 0 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}}$.

So our 1st way is with eigenvalue method ✓

$\boxed{2^{nd}\text{ way}}$ Each Row Reads as

$$x_1' = 2x_1 + x_2$$
$$x_2' = 2x_2$$
$$x_3' = 2x_3$$

We can solve for x_3, x_2; then x_1.

$\underline{x_3:}$ Says $\frac{dx_3}{dt} = 2x_3 \iff \frac{dx_3}{x_3} = 2dt$; $\ln|x_3| = 2t + C$; $\boxed{x_3 = A_3 e^{2t}}$

$\underline{x_2:}$ Is the same eqn but in x_2 as $\boxed{x_2 = A_2 e^{2t}}$

$\underline{x_1:}$ Rewrite as $x_1' - 2x_1 = A_2 e^{2t}$ now. Treat like 2^{nd} order,

$\underline{x_{1,c}} \Rightarrow r - 2 = 0$; $r = 2$, $\boxed{x_{1,c} = A_1 e^{2t}}$.

$\underline{x_{1,p}} \Rightarrow$ Guess $\beta t e^{2t}$ because e^{2t} is a complementary piece.

$x_{1,p} = \beta t e^{2t}$; $x_{1,p}' = \beta e^{2t} + 2\beta t e^{2t}$.

plugin $\Rightarrow \beta e^{2t} + \cancel{2\beta t e^{2t}} - \cancel{2\beta t e^{2t}} = A_2 e^{2t}$; $\boxed{\beta = A_2}$.

So, $\boxed{x_1 = x_{1,c} + x_{1,p} = A_1 e^{2t} + A_2 t e^{2t}.}$

From 1^{st} way, had

$$x_1 = C_2 e^{2t} + C_3 t e^{2t}$$
$$x_2 = C_3 e^{2t}$$
$$x_3 = C_1 e^{2t}$$

$\Rightarrow$ $\underline{\text{Same as } 1^{st} \text{ way}}$,

call $A_3 \leftrightarrow C_1$.
$A_2 \leftrightarrow C_3$.
$A_1 \leftrightarrow C_2$. ✓

$\underline{\text{Lastly}}$: Complete all the other problems.

3.3 # 1–5,
3.4 # 6–11
3.7 # 2–7

(−1) each incomplete or way off.

5 pts

Math 3D Quiz 3 Morning - February 23rd

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Show all of your work. *There is a question on the back side.*

1. [10pts] Find the general solution to $\vec{y}' = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix} \vec{y}$ using the eigenvalue method.