# MATH 117, DYNAMICAL SYSTEMS
# SAMPLE MIDTERM

## Problem 1.

Let $f : [0,1] \to [0,1]$ be a homeomorphism (i.e. $f$ is a continuous bijection and $f^{-1}$ is also continuous) such that $f(0) = 1$ and $f(1) = 0$. Prove that $f$ has exactly one fixed point.

## Problem 2.

Suppose that each of the rotations of the circle $R_\alpha : S^1 \to S^1$ and $R_\beta : S^1 \to S^1$ is transitive. Does it imply that their product $f = R_\alpha \times R_\beta : \mathbb{T}^2 \to \mathbb{T}^2$ is also transitive? Explain your answer.

## Problem 3.

Prove that $\sup_{n\in\mathbb{Z}}(\sin n) = 1$.

## Problem 4.

Compute the topological entropy of the endomorphism of the torus

$$f : \mathbb{T}^2 \to \mathbb{T}^2, \;\; f(x,y) = (3x, 5y) (\operatorname{mod} 1).$$

## Problem 5.

Show that composition of two orientation-reversing homeomorphisms of the circle is an orientation-preserving homeomorphism.