MATH 117, DYNAMICAL SYSTEMS SAMPLE FINAL

Problem 1.

Give an example of a topological dynamical system $f : X \to X$ such that

a) f is minimal but not topologically mixing;

b) f is topologically mixing but not minimal.

Problem 2.

TRUE OR FALSE: If $f, g : S^1 \to S^1$ are homeomorphisms of the circle with rotation numbers $\rho(f) = \alpha$ and $\rho(g) = \beta$ then $\rho(f \circ g) = \alpha + \beta$. Prove or give a counterexample.

Problem 3.

Show that the flow in $\mathbb{R}^2$ given (in polar coordinates) by

$$\begin{cases} \dot{r} = \frac{r(1-r)}{1+r^4} \\ \dot{\theta} = 1 \end{cases}$$

has a periodic orbit. Is this periodic orbit attracting or repelling? Explain your answer.

Problem 4.

Consider the topological Markov chain given by the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$. How many periodic point of (smallest) period 4 does it have?

Problem 5.

Consider the topological Markov chain given by the matrix $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$. Let P_n be the number of its periodic orbits of (not necessarily smallest) period n. Prove that

$$\lim_{n \to \infty} \frac{1}{n} \log P_n = \log \frac{1 + \sqrt{5}}{2}.$$