DYNAMICAL SYSTEMS, MATH 117, HW#5

Problem 1.

Consider the map $f : [0,1] \to [0,1]$, $f(x) = \begin{cases} 1/2 + x, & \text{if } x \in [0, 1/2]; \\ 2 - 2x, & \text{if } x \in [1/2, 1]. \end{cases}$
Periodic points of what periods does this map have?

Problem 2.

Suppose a homeomorphism of the circle $f : S^1 \to S^1$ has a periodic point of period 3. Can it have a periodic point of period 7? Explain your answer.

Problem 3.

Determine whether there exists a homeomorphism f of the circle such that $f \circ E_2 = E_4 \circ f$. Explain your answer. (Here $E_m : S^1 \to S^1,\ E_m(x) = mx(mod\, 1)$.)

Problem 4.

Consider the set C of all points of the unit interval which have a binary representation without two successive zeros. Prove that C is an uncountable set invariant under the map $T : [0,1) \to [0,1)$, $T(x) = 2x(\text{mod } 1)$.

Problem 5.

Consider the map $f : \mathbb{T}^2 \to \mathbb{T}^2$, $T(x,y) = (x + \alpha, x + y)(\text{mod } 1)$, where $\alpha \notin \mathbb{Q}$. Is it transitive? topologically mixing?