MATH 117, DYNAMICAL SYSTEMS HOMEWORK #1

Chapter 3, problems 7 and 11, and the following problems:

Problem 1.

Let $f : [0, 1] \to [0, 1]$ be a homeomorphism (i.e. f is a continuous bijection and f^{-1} is also continuous) such that $f(0) = 1$ and $f(1) = 0$. Prove that f has exactly one fixed point.

Problem 2.

Suppose that each of the rotations of the circle $R_\alpha : S^1 \to S^1$ and $R_\beta : S^1 \to S^1$ is minimal (i.e. every orbit is dense in the phase space). Does it imply that their product $f = R_\alpha \times R_\beta : \mathbb{T}^2 \to \mathbb{T}^2$ is also minimal? Explain your answer.

Problem 3.

Prove that $\sup_{n\in\mathbb{Z}}(\sin n) = 1$.