MATH 117, DYNAMICAL SYSTEMS
SAMPLE FINAL

Problem 1.

Consider the map $f : \mathbb{R}^1 \to \mathbb{R}^1$ given by

$$f(x) = \begin{cases} 3x, & \text{if } x \leq 1/3; \\ 1, & \text{if } 1/3 < x < 2/3; \\ 3 - 3x, & \text{if } 2/3 \geq x. \end{cases}$$

a) What is the set of points with bounded orbits under the iterations of f?

b) Describe the set $S = \{x \in \mathbb{R} \mid f^n(x) = 0 \text{ for some } n \in \mathbb{N}\}$.

Problem 2.

Find the Hausdorff dimension of Cantor set generated by the contractions

$$f_1(x) = \frac{x}{25} \text{ and } f_2(x) = \frac{1}{2} + \frac{x}{5}.$$

Problem 3.

Prove that a homeomorphism of $\mathbb{R}$ cannot have periodic points with prime period greater than 2.

Problem 4.

Let X be a compact metric space. A point p is a non-wandering point for a continuous map $f : X \to X$ if for any open set $U, p \in U$, there exists $x \in U$ and $n > 0$ such that $f^n(x) \in U$ (note that we do not require that p itself return to U). Let $\Omega(f)$ denote the set of all non-wandering points for f.

a) Prove that $\Omega(f)$ is a closed set.

b) Prove that $\Omega(f)$ is an invariant set.

Problem 5.

Give an example of a topological Markov chain that has a dense set of periodic points but is not topologically mixing.