Math 117, Dynamical Systems
Sample Midterm

Problem 1.

Consider a map $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^3 - 3x^2 + 3x$

a) Find all the fixed points of f and determine which of them are attracting or repelling;

b) Describe the set $B = \{x \in \mathbb{R} \mid \{f^n(x)\}_{n\in\mathbb{N}} \text{ is bounded}\}$?

Problem 2.

Find $\sup_{t\in\mathbb{R}}(\cos t + \sin\sqrt{3}t)$.

Problem 3.

Consider the following map of the torus $\mathbb{T}^2 = \mathbb{R}^2 \backslash \mathbb{Z}^2$:

$$f : \mathbb{T}^2 \to \mathbb{T}^2,\ f(x, y) = (2x, 3y)(\operatorname{mod} 1).$$

Prove that f is topologically mixing and periodic points of f are dense in $\mathbb{T}^2$. Find the the number of its periodic points of (not necessarily smallest) period n for each $n \in \mathbb{N}$.

Problem 4.

Give an example of a topological dynamical system $f : X \to X$ such that

a) f is minimal but not topologically mixing;

b) f is topologically mixing but not minimal.

Problem 5.

Consider the topological Markov chain given by the matrix $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$. Let P_n be the number of its periodic points of (not necessarily smallest) period n. Prove that

$$\lim_{n\to\infty} \frac{1}{n} \log P_n = \log \frac{1+\sqrt{5}}{2}.$$