# Final Sample

Problem 1.

Consider the map $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + \frac{1}{6}$.

a) Check that $x = 1$ is a fixed point of $f$. Is it attracting, repelling, or neutral?

b) Find other fixed points of $f$, and determine whether they are attracting, repelling, or neutral.

Problem 2.

Consider the topological Markov chain given by the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}.$$

Find the number of periodic points of (not necessarily smallest) period $m$ for each $m \in \mathbb{N}$.

Problem 3.

Let $S \subseteq \mathbb{R}^2$ be a compact subset of the plane such that

$$S = f_1(S) \cup f_2(S) \cup f_3(S),$$

where the contractions $f_1, f_2, f_3$ are defined by

$$f_1(x,y) = \left(\frac{x}{5}, \frac{y}{5}\right), f_2(x,y) = \left(\frac{x}{5} + 1, \frac{y}{5}\right), f_3(x,y) = \left(\frac{x}{5}, \frac{y}{5} + 1\right).$$

Find the box counting dimension of $S$.

Problem 4.

a) Is it possible for a continuous map $f : \mathbb{R} \to \mathbb{R}$ to have a periodic point of (smallest) period $7$, but not a periodic point of (smallest) period $16$?

b) Is it possible for a continuous map $f : \mathbb{S}^1 \to \mathbb{S}^1$ to have a periodic point of (smallest) period $7$, but not a periodic point of (smallest) period $16$?

Problem 5.

Consider the family of maps $f_a : \mathbb{R} \to \mathbb{R}$, $f_a(x) = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + a$. Show that for any value of the parameter $a \in \mathbb{R}$ the map $f_a$ cannot have more than two attracting periodic orbits.