DYNAMICAL SYSTEMS, MATH 117, HW#5

Exercises 11.2, 11.4, 11.6, 11.7, and the following problems:

Problem 1.

Consider the map $f : [0,1] \rightarrow [0,1]$, $f(x) = \begin{cases} 1/2 + x, & \text{if } x \in [0,1/2]; \\ 2 - 2x, & \text{if } x \in [1/2,1]. \end{cases}$ Periodic points of what periods does this map have?

Problem 2.

Let $\sigma_A : \Sigma_A \to \Sigma_A$ be a topological Markov chain with the matrix

$$A = \left(\begin{array}{rrr} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array} \right) \,.$$

Find $\lim_{n\to\infty} \frac{1}{n} \log (\#Per_n(\sigma_A))$, where $\#Per_n(\sigma_A)$ is the number of periodic points of (not necessarily minimal) period *n*.

Problem 3.

a) Is it possible to find a continuous map $f : [0, 1] \rightarrow [0, 1]$ such that f^2 has a periodic point of prime period 20, but f^3 has no periodic points of prime period 34?

b) Is it possible to find a continuous map $f : [0,1] \rightarrow [0,1]$ such that f^2 has a periodic point of prime period 15, but f^3 has no periodic points of prime period 28?