MATH 117, DYNAMICAL SYSTEMS
SAMPLE FINAL

Problem 1.

Find all the fixed points of the map $T : \mathbb{R}^3 \to \mathbb{R}^3$, $T(x, y, z) = (2xy - z, x, y)$.

Problem 2.

For each of the following statements determine whether it is true or false (explain your answer).

a) If a continuous map $f : S^1 \to S^1$ has a periodic point of prime period 3, then it has periodic points of all prime periods.

b) If a continuous map $f : \mathbb{R}^2 \to \mathbb{R}^2$ has a periodic point of prime period 3, then it has periodic points of all prime periods.

c) If a continuous map $f : S^1 \to S^1$ has a fixed point and a periodic point of prime period 3, then it has periodic point of prime period 2.

Problem 3.

Consider a family of maps $G_c : \mathbb{C} \to \mathbb{C}$, $G_c(z) = z^3 + c$. Show that if $|c| > 2$, then $G_c^n(0) \to \infty$ as $n \to +\infty$. Can one replace the condition $|c| > 2$ by the condition $|c| > 1.5$ in this statement?

Problem 4.

Consider the map $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \frac{x^3}{3} - \frac{x^2}{2}$.

a) Show that $Sf < 0$. Find the number of critical points of f. Conclude that f has at most 2 attracting periodic orbits.

b) Find all the attracting periodic orbits of f.

Problem 5.

Let $C \subset [0,1]$ be the Cantor set generated by contractions

$$\phi_1 : x \mapsto \frac{x}{3},\ \phi_2 : x \mapsto \frac{x}{9} + \frac{1}{2},\ \phi_3 : x \mapsto \frac{x}{9} + \frac{8}{9}.$$

Find its box counting dimension.