MATH 117, DYNAMICAL SYSTEMS
SAMPLE MIDTERM

Problem 1.

Consider a map $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^3 - 3x^2 + 3x$

a) Find all the fixed points of f and determine which of them are attracting or repelling;

b) Describe the set $B = \{x \in \mathbb{R} \mid \{f^n(x)\}_{n \in \mathbb{N}} \text{ is bounded}\}$.

Problem 2.

Suppose that $f : X \to X$ is a topologically transitive homeomorphism. Does it imply that for every $n \in \mathbb{N}$ the map $f^n : X \to X$ is also topologically transitive?

Problem 3.

Consider the following map of the torus $\mathbb{T}^2 = \mathbb{R}^2 \backslash \mathbb{Z}^2$:

$$f : \mathbb{T}^2 \to \mathbb{T}^2,\ f(x, y) = (2x, 3y)(\text{mod } 1).$$

Find the the number of its periodic points of (not necessarily smallest) period n for each $n \in \mathbb{N}$.

Problem 4.

Give an example of a topological dynamical system $f : X \to X$ such that

a) f is transitive but not chaotic;

b) f has dense set of periodic points but not chaotic;

c) f has sensitive dependence on initial conditions but not chaotic.

Problem 5.

Let Σ be the metric space of all sequences of zeros and ones, and $\tilde{\Sigma} \subset \Sigma$ be the subset that consists of all sequences that do not have more than five zeros in a row. Which of the following statements are true?

a) $\tilde{\Sigma}$ is dense in Σ;

b) $\tilde{\Sigma}$ is invariant under the topological Bernoulli shift $\sigma : \Sigma \to \Sigma$;

c) $\tilde{\Sigma}$ is a closed subset of Σ;

d) $\tilde{\Sigma}$ is an open subset of Σ;

e) $\tilde{\Sigma}$ contains infinitely many periodic points of $\sigma : \Sigma \to \Sigma$.