Final Sample

Problem 1.

Consider the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$.

a) Find eigenvalues of A;

b) Find eigenvectors of A;

c) Find an explicit formula for A^N.

Problem 2.

Prove that if $A \in M_{n\times n}$ is diagonalizable, then A^3 is also diagonalizable. Does the converse statement hold?

Problem 3.

Let $A \in M_{n\times n}(\mathbb{R})$, $n \in \mathbb{N}$, be a matrix with the entries $A_{ij} = (-1)^{i+j}$. Find $det(A)$.

Problem 4.

Let $p(t) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ be the characteristic polynomial of a matrix $A \in M_{3\times 3}(\mathbb{R})$. Show that

a) $a_3 = -1$;

b) $a_0 = \ det\,(A)$;

c) $a_2 = Tr(A)$.

Problem 5.

For each of the following statements answer whether it is true or false (explain your answer):

1) In $M_{n\times n}(\mathbb{R})$ the set of matrices with zero determinant is a subspace.

2) In $M_{n\times n}(\mathbb{R})$ the set of matrices with zero trace is a subspace.

3) In $M_{n\times n}(\mathbb{R})$ the set of diagonal matrices is a subspace.

4) In $M_{n\times n}(\mathbb{R})$ the set of diagonalizable matrices is a subspace.

5) In $M_{n\times n}(\mathbb{R})$ the set of matrices of rank one is a subspace.