Midterm Sample

Problem 1.

Write down a basis in the space of symmetric 2×2 matrices.

Problem 2.

Let a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a reflection in the line $x = y$. Prove that T is linear and find its matrix representation (with respect to the standard basis in $\mathbb{R}^2$).

Problem 3.

Suppose V is a finite dimensional vector space, W_1, W_2 are subspaces, and $W_1 + W_2 = V$. Prove that

$$\dim V = \dim W_1 + \dim W_2 - \dim (W_1 \cap W_2).$$

Problem 4.

TRUE or FALSE: There exists a basis $\{p_1, p_2, p_3, p_4\}$ in P_3 such that none of the polynomials p_1, p_2, p_3, p_4 has degree 2.

Problem 5.

Find nullity and rank of the linear transformation $T : M_{3\times3} \to M_{3\times3}$ given by $T(A) = A + A^t$.

Reminder: if $A = (a_{ij})$, then $A^t = (b_{ij})$ with $b_{ij} = a_{ji}$.