INTRODUCTION TO TOPOLOGY, MATH 141, HW#5

Problem 1.

Consider $C[0,1]$ - the space of all continuous function on $[0,1]$ equipped with the norm $\|\cdot\|_\infty$. Let $P \subset C[0,1]$ be the space of all polynomials.

a) Show that P is a linear subspace of $C[0,1]$;

b) Show that P is not closed in $C[0,1]$;

c) Show that P is not open in $C[0,1]$.

Problem 2.

Consider

$$C^1[0,1] = \left\{ f : [0,1] \to \mathbb{R} \mid f \text{ is differentiable and } f' \text{ is continuous} \right\}.$$

a) Show that $C^1[0,1]$ is a linear subspace of $C[0,1]$;

b) Equip both $C^1[0,1]$ and $C[0,1]$ with $\|\cdot\|_\infty$ norm. Show that $C^1[0,1]$ is not closed and not open in $C[0,1]$;

c) Define $T : C^1[0,1] \to C[0,1]$ as $T(f) = f'$. Prove that T is a linear operator. Is it bounded?