Introduction to Topology, Math 141, HW#7

Problem 1.

Prove that the open upper half plane

$$\{(x,y) \in \mathbb{R}^2 \mid y > 0\}$$

is homeomorphic to the unit ball

$$\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}.$$

Problem 2.

Prove that the punctured space

$$\mathbb{R}^3 \backslash \{(0,0,0)\}$$

is homeomorphic to the exterior of the closed unit ball

$$\{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 > 1\}.$$

Problem 3.

Let X be a topological space with discrete topology. Denote by $\mathcal{B}$ the collection of sets that contain not more than one point. Prove that $\mathcal{B}$ is a base of open sets for X. Show that any other base of open sets must contain all elements of $\mathcal{B}$.

Problem 4.

Prove that in a Hausdorff topological space a sequence cannot converge to more than one point.

Problem 5.

Suppose that X is a T_1-space, and $Y \subseteq X$ is a subspace (equipped with the relative topology). Prove that Y is also a T_1-space.