MATH 141, INTRODUCTION TO TOPOLOGY SAMPLE FINAL

Problem 1.

Let d_1 and d_2 be two different metrics on the same set M.

a) Is the function $d_+(x, y) = d_1(x, y) + d_2(x, y)$ a metric?

b) Is the function $d_*(x, y) = d_1(x, y) \cdot d_2(x, y)$ a metric?

c) Is the function $d_{max}(x, y) = \max(d_1(x, y), d_2(x, y))$ a metric?

d) Is the function $d_{min}(x, y) = \min(d_1(x, y), d_2(x, y))$ a metric?

Problem 2.

a) Give a definition of a topological space.

b) Define

$$\mathcal{T} = \left\{ A \subseteq \mathbb{R}^2 \mid \text{for any horizontal line } \ l \subset \mathbb{R}^2 \text{ the intersection } A \cap l \text{ is open in } l \right\}.$$

Does $\mathcal{T}$ define topology in $\mathbb{R}^2$? Is it the same topology as the standard one?

Problem 3.

Let (M, d) be a metric space. Suppose that open sets $U, V \subseteq M$ are dense in M. Show that $U \cap V$ is also dense in M. Can one replace in this statement the metric space by an arbitrary topological space?

Problem 4.

A subset $A \subseteq \mathbb{R}^n$ is *polyline-connected* if any two points of A are joined by a finite broken line (a polyline) contained in A. Prove that an open set in $\mathbb{R}^n$ is polyline-connected if and only if it is path connected.

Problem 5.

Show that any finite set in a Hausdorff topological space is closed.