# Real Analysis
# Math 205A, Fall 2015

**Midterm Sample**

Problem 1.

Prove that for any real numbers $x, y \in \mathbb{R}$ the sequence $\{a_n\}_{n\in\mathbb{N}}$ defined by

$$a_1 = x, a_2 = y, a_{n+2} = \frac{a_n + a_{n+1}}{2}, n \geq 1,$$

converges. Find the limit.

Problem 2.

Prove that any two open subsets of $\mathbb{R}^2$ are equivalent (i.e. have the same cardinality).

Problem 3.

a) Is it true that for any continuous function $f : D \to \mathbb{R}$, where $D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$, there exists a continuous function $F : \mathbb{R}^2 \to \mathbb{R}$ such that $F|_D = f$?

b) Is it true that for any subset $A \subset M$ of a metric space $M$, and for any continuous function $f : A \to \mathbb{R}$ there exists a continuous function $F : M \to \mathbb{R}$ such that $F|_A = f$?

Problem 4.

Suppose that $M$ is a metric space such that any proper subset $A \subset M$ is contained in some larger connected subset of $M$. Prove that $M$ is connected.

Problem 5.

Is it possible for a continuous function $f : \mathbb{R}^2 \to \mathbb{R}$ to have uncountably many strict local minima?