Real Analysis
Math 205B/H140B, Winter 2016

Homework 8, due March 8, 2016 in class

Chapter 15, problems 2, 3, 4, 5, 6, and the following problems:

Problem 1.

Find an orthonormal basis for the space of first degree polynomials, with inner product

$$< f, g >= \int_{-1}^{1} f(x)g(x)dx$$

Problem 2.

Use the previous problem to determine the constants a, b that minimize the integral

$$\int_{-1}^{1} |e^x - ax - b|^2 dx$$

Problem 3.

Find the Fourier series of the function f defined by

$$f(x) = \begin{cases} -1, & \text{if } -\pi < x < 0; \\ 1, & \text{if } 0 < x < \pi, \end{cases}$$

and f has period 2π. What does the Fourier series converge to at $x = 0$?

Problem 4.

Let $f(x) = x$ for $x \in (-\pi, \pi]$, and $f(x + 2\pi) = f(x)$. Find the Fourier series of f.

Problem 5.

Let α be any real number other than an integer. Let $f(x) = \cos(\alpha x)$ for $x \in (-\pi, \pi]$, and $f(x + 2\pi) = f(x)$. Prove that Fourier series of f is given by

$$\frac{\sin \alpha\pi}{\alpha\pi} + \frac{1}{\pi} \sum_{k=1}^{\infty} \left[\frac{\sin(\alpha + k)\pi}{\alpha + k} + \frac{\sin(\alpha - k)\pi}{\alpha - k} \right] \cos(kx)$$