REAL ANALYSIS
MATH 205C/H140C, SPRING 2016

Homework 1, due April 8, 2016 in class

Problem 1.

The pressure in the space at the position (x, y, z) is $p(x, y, z) = x^2+y^2-z^3$ and the trajectory of an observer is the curve $\bar{r}(t) = (t, t, 1/t)$. Using the chain rule, compute the rate of change of the pressure the observer measures at time $t = 2$.

Problem 2.

Find the directional derivative $Df(x, \bar{u})$, where $x = (1, 2)$, $\bar{u} = (1, 1)$, and $f : \mathbb{R}^2 \to \mathbb{R}^1$, $f(x, y) = x^5y + y^3 + x + y$.

Problem 3.

Let $f : \mathbb{R}^2 \to \mathbb{R}^1$ be defined by

$$f(x,y) = \begin{cases} \frac{x|y|}{\sqrt{x^2+y^2}}, & \text{if } (x,y) \neq 0; \\ 0, & \text{if } (x,y) = 0. \end{cases}$$

Is f differentiable at $(0, 0)$? Explain your answer.

Problem 4.

Let $f : \mathbb{R}^2 \to \mathbb{R}^1$ be defined by $f(x, y) = \sqrt{|xy|}$. Prove that f is not differentiable at $(0, 0)$.

Problem 5.

Let $f : \mathbb{R}^2 \to \mathbb{R}^1$ be a function such that $|f(x)| \le |x|^2$. Show that f is differentiable at $\bar{0}$.

Problem 6.

Let $g : \mathbb{R} \to \mathbb{R}$ be a continuous function. Define $f(x, y) = \int_0^{x+y} g(t)dt$. Prove that f is differentiable and find $Df(x, y)$.

Problem 7.

Let $g : \mathbb{R} \to \mathbb{R}$ be a continuous function. Define $f(x, y) = \int_0^{x \cdot y} g(t)dt$. Prove that f is differentiable and find $Df(x, y)$.

Problem 8.

Let $g : \mathbb{R} \to \mathbb{R}$ be a continuous function. Define $f(x, y) = \int_{x+y}^{x \cdot y} g(t)dt$. Prove that f is differentiable and find $Df(x, y)$.

Problem 9.

Let $f : \mathbb{R}^2 \to \mathbb{R}^1$ be defined by

$$f(x,y) = \begin{cases} xy\frac{x^2-y^2}{x^2+y^2}, & \text{if } (x,y) \neq 0; \\ 0, & \text{if } (x,y) = 0. \end{cases}$$

Show that $D_{1,2}f(0,0) \neq D_{2,1}f(0,0)$. Does it contradict to the result we obtained?

Problem 10.

Let $f : \mathbb{R}^2 \to \mathbb{R}^1$ be defined by

$$f(x,y) = \begin{cases} (x^2+y^2)\sin\frac{1}{\sqrt{x^2+y^2}}, & \text{if } (x,y) \neq 0; \\ 0, & \text{if } (x,y) = 0. \end{cases}$$

Show that f is differentiable but $f \notin C^1(\mathbb{R}^2)$.