Real Analysis
Math 205C/H140C, Spring 2016

Homework 5, due May 10, 2016 in class

Problem 1.

TRUE or FALSE: If a linear transformation $A : \mathbb{R}^n \to \mathbb{R}^n$ preserves volumes then A is an isometry.

Problem 2.

TRUE or FALSE: If a linear transformation $A : \mathbb{R}^n \to \mathbb{R}^n$ is orthogonal (i.e. $A^{tr}A = I_n$) then it sends any orthonormal basis to an orthonormal basis.

Problem 3.

Consider the vectors $\bar{v}, \bar{u} \in \mathbb{R}^2$, $\bar{v} = (\cos\theta, \sin\theta)$, $\bar{u} = (\cos(2\theta), \sin(2\theta))$. For which θ this pair of vectors form a basis in $\mathbb{R}^2$? For which θ this basis is oriented in the same way as the standard basis? In the opposite way?

Problem 4.

Evaluate the integral $\int_{\mathbb{R}^2} f(x,y)$, where

$$f(x,y) = e^{-(x^2+2(x-y)^2+y^2)}$$

Problem 5.

Find the volume of the set $\{(x,y,z) \mid x^2 + 2y^2 + 3z^2 \leq 1\} \subset \mathbb{R}^3$.

Problem 6.

Evaluate

$$\int_R \left(\frac{x-y}{x+y+2}\right)^2,$$

where $R \subset \mathbb{R}^2$ is a square with the vertices $(1,0), (0,1), (-1,0)$, and $(0,-1)$.

Problem 7.

Evaluate $\int_R (x^2+y)$, where R is an annular region between the two circles $x^2+y^2 = 1$ and $x^2+y^2 = 4$.

Problem 8.

Evaluate

$$\int_R (x+y)^2 \sin^2(x-y),$$

where $R \subset \mathbb{R}^2$ is a square with the vertices $(1,0), (0,1), (1,2)$, and $(2,1)$.

Problem 9.

Evaluate $\int_R f(x,y,z)$, where $f(x,y,z) = x^2+y^2+z^2$ and R is an upper half sphere $\{x^2+y^2+z^2 < 1, z > 0\}$. *Hint: use spherical coordinates,* $x = r\sin\phi\cos\theta, y = r\sin\phi\sin\theta, z = r\cos\phi$.

Problem 10.

Suppose $q(x)$ is a continuous function on $[0,1]$. Prove that

$$\int_0^1 \left[q(x)\left(\int_0^x q(t)dt\right)\right] dx = \frac{1}{2}\left(\int_0^1 q(x)dx\right)^2$$