Real Analysis
Math 205C/H140C, Spring 2016

Homework 7, due June 3, 2016 in class

Problem 1.

Formulate the divergence theorem (in terms of vector calculus) in the case of a bounded domain with a smooth boundary in $\mathbb{R}^n$ (instead of $\mathbb{R}^3$).

Problem 2.

Use Green's Theorem to compute the area inside the Lemniscate of Gerono, which has equation $x^4 = x^2 - y^2$.

Problem 3.

Evaluate

$$\int_\gamma y^4 dx + 2xy^3 dy,$$

where γ is an ellipse $x^2 + 2y^2 = 2$.

Problem 4.

Determine the area enclosed by cycloid, i.e. the curve parameterized by

$$\gamma(t) = (t - \sin t, 1 - \cos t), \ \ t \in [0, 2\pi],$$

and the x-axis.

Problem 5.

Let E be the solid unit cube with opposing corners at the origin and $(1, 1, 1)$ and faces parallel to the coordinate planes. Let S be the boundary surface of E, oriented with the outward-pointing normal. If the vector field $\bar{F}$ is given by

$$\bar{F} = (2xy, 3ye^z, x \sin z),$$

find $\int_S < \bar{F}, \bar{n} > dA$ using the divergence theorem.

Problem 6.

TRUE or FALSE: If $S \subset \mathbb{R}^3$ is a smooth closed surface (that is, a surface without a boundary curve), then for any smooth vector field $\bar{F}$ in $\mathbb{R}^3$ we have

$$\int_S < \text{curl}\, \bar{F}, \bar{n} > dA = 0.$$

Problem 7.

Evaluate the integral

$$\int_\gamma y^3 dx + xzdy + x^3 dz,$$

where γ is the intersection of the plane $y + z = 1$ with the cylinder $x^2 + y^2 = 1$, oriented counterclockwise when viewed from above.

Problem 8.

An electric charge at $\bar{0} \in \mathbb{R}^3$ generates the field $\bar{E} = \frac{\bar{x}}{\|\bar{x}\|^3}$. What is the flux of $\bar{E}$ through the unit sphere? What is the flux of $\bar{E}$ through any other sphere containing the origin $(0, 0, 0)$ inside?

Problem 9.

Let S be the upper hemisphere of the unit sphere $x^2 + y^2 + z^2 = 1$. Use Stokes' theorem to evaluate $\int_S < \bar{F}, \bar{n} > dA$, where the vector field $\bar{F}$ is given by

$$\bar{F} = (x^3 e^y, -3x^2 e^y, 0),$$

and $\bar{n}$ is the normal vector pointing upward.

Problem 10.

Let γ be the curve of the intersection of the surface $2x^2 + 2y^2 + z^2 = 9$ with $z = \frac{1}{2}\sqrt{x^2 + y^2}$, oriented counterclockwise when viewed from above. Evaluate

$$\int_\gamma 2y dx + 2yz dy + (xz^3 + y \sin(z^2)) dz$$