Complex Analysis, HW # 6

Chapter 7, problems 30, 31, **and these problems:**

Problem 1.

For $\alpha \in \mathbb{R}$ let $L_\alpha = \{re^{i\alpha} \mid r \geq 0\}$. Suppose that $0 < \alpha < 2\pi$. Show that if α/π is rational then there exists a non-trivial function u harmonic in $\mathbb{C}$ which vanishes on L_0 and L_α.

Problem 2.

For $\alpha \in \mathbb{R}$ let $L_\alpha = \{re^{i\alpha} \mid r \geq 0\}$. Suppose that $0 < \alpha < 2\pi$. Show that if α/π is irrational then any harmonic in $\mathbb{C}$ function that vanishes on L_0 and L_α must vanish identically.

Problem 3.

Suppose f is entire, $f(x)$ is real for all $x \in \mathbb{R}$ and $f(iy)$ is purely imaginary for all $y \in \mathbb{R}$. Show that $f(-z) = -f(z)$.

Problem 4.

Let s be a real number, and let the function u be defined in $\mathbb{C}\backslash(-\infty, 0]$ by

$$u(re^{i\theta}) = r^s \cos s\theta \quad (r > 0,\ -\pi < \theta < \pi).$$

Prove that u is a harmonic function.

Problem 5.

Let f be an entire function which is real valued on the unit circle. Prove that f is constant.