Complex Analysis Math 220A

Midterm Sample Exam

Problem 1.

How many roots of the equation $z^4 + z^3 - 4z + 1 = 0$ are in the ring $1 < |z| < 4$?

Problem 2.

Construct a conformal mapping that sends the open set

$$U_1 = \{z = x + iy \mid 0 < y < x\}$$

to the open set

$$U_2 = \{z = x + iy \mid x^2 + y^2 < 1,\ y > 0\}.$$

Problem 3.

Let u be a harmonic function on $\mathbb{R}^2$ that does not take zero value (i.e. $u(x) \neq 0 \ \ \forall x \in \mathbb{R}^2$). Show that u is constant.

Problem 4.

Describe the group of conformal automorphisms of the domain

$$U = \left\{ |z| < 1,\ z \notin \left\{ \frac{1}{2}, -\frac{1}{2} \right\} \right\}.$$

Problem 5.

Let f be a holomorphic function that maps the unit disc to the unit disc. Assume that for some $a \neq b$, $|a| < 1, |b| < 1$, we have $f(a) = f(b) = 0$. Show that for any z from the unit disc we have

$$|f(z)| \leq \left| \frac{z - a}{1 - z\bar{a}} \right| \cdot \left| \frac{z - b}{1 - z\bar{b}} \right|$$