Complex Analysis Math 220B

Final Exam (sample)

Problem 1.

Let f be a bounded holomorphic function in the upper half plane and continuous on its closure. Suppose also that $f(z)$ is real for real z. Prove that f must be constant.

Problem 2.

Prove that the product $\prod_{k=1}^{\infty}\left(\frac{z^n}{n!}+\exp\left(\frac{z}{2^n}\right)\right)$ converges uniformly on compact sets to an entire function.

Problem 3.

Let $\mathbb{H}=\{z\in\mathbb{C}:\text{Im } z>0\}$ and $f:\mathbb{H}\to\mathbb{H}$ be analytic. Prove that

$$\left|\frac{f(z)-f(i)}{f(z)-\overline{f(i)}}\right|\le\left|\frac{z-i}{z+i}\right|,\quad z\in\mathbb{H}.$$

Problem 4.

Prove that there is no one-to-one holomorphic map from the unit disk $\mathbb{D}=\{|z|<1\}$ onto the punctured disk $\mathbb{D}\backslash\{0\}=\{0<|z|<1\}$. Prove also that there is a holomorphic mapping of the unit disk $\mathbb{D}=\{|z|<1\}$ onto the punctured disk $\mathbb{D}\backslash\{0\}=\{0<|z|<1\}$.

Problem 5.

Let $\mathbb{D}$ be the unit disc, and suppose that $f:\mathbb{D}\to\mathbb{D}\backslash\{0\}$ is analytic and $f(0)=\frac{1}{2}$. Prove that $\left|f\left(\frac{1}{2}\right)\right|\ge\frac{1}{8}$.