Complex Analysis

Practice Qualifying Exam

Monday, June 1, 2015 — 1:00pm - 3:30pm

This Exam is for training purposes only. It will not influence you Math 220C final grade, and cannot substitute the actual Qualifying Exam in Complex Analysis in any way.

Problem	1	2	3	4	5	6	7	8	Σ
Points									

Student's name:

Problem 1.

Prove that there is no function f such that f is analytic on the punctured unit disc $\mathbb{D} \setminus \{0\}$, and f' has a simple pole at 0.

Problem 2.

Prove that the zeros of the polynomial $p(z) = z^n + c_{n-1}z^{n-1} + \ldots + c_1 z + c_0$ all lie in the open disk with center 0 and radius

$$R = \sqrt{1 + |c_0|^2 + |c_1|^2 + \ldots + |c_{n-1}|^2}.$$

Problem 3.

Find explicitly a conformal mapping of a domain $\mathbb{D}\backslash(-1, 1/4]$ to the unit disc $\mathbb{D}$.

Problem 4.

Evaluate $\int_0^\infty \frac{(\log x)^2}{1+x^2} dx.$

Problem 5.

Let f be a bounded analytic function in the upper half-plane $\mathbb{H}$. Suppose that $f(in) = e^{-n}$ for all $n \in \mathbb{N}$. Find $f(1+i)$. (You need to explain why the value that you found is the only possible.)

Problem 6.

Let $\mathcal{F}$ be the family of all analytic functions

$$f(z) = z + a_2 z^2 + a_3 z^3 + \dots$$

on the open unit disc, such that $|a_n| \leq n$ for each n. Prove that $\mathcal{F}$ is a normal family.

Problem 7.

Let f be an entire non-constant function that satisfies the functional equation $f(1-z) = 1 - f(z)$ for all $z \in \mathbb{C}$. Show that $f(\mathbb{C}) = \mathbb{C}$.

Problem 8.

Determine the complex numbers z for which the power series

$$\sum_{n=1}^{\infty} \frac{z^n}{n^{2\log n}}$$

and its term by term derivatives of all orders converge absolutely.