# Complex Analysis, HW # 1

## Problem 1.

Find the integral

$$\int_0^\infty \frac{\cos(2x)}{1+x^2}dx.$$

## Problem 2.

Consider a series $f(z) = \sum_{n=0}^\infty \frac{z^n}{n+1}$.

a) What is the radius of convergence of this series?

b) Take the power series for $f$ near the point $\frac{i}{2}$, i.e. $f(z) = \sum_{n=0}^\infty a_k(z-\frac{i}{2})^k$. What is the radius of convergence of this series?

## Problem 3.

Let $f(z) = \sum_{k=0}^\infty a_k z^k$ be an entire function. Find the integral

$$\int_{|z|=1} f\left(\sin\left(\frac{1}{z}\right)\right) dz$$

## Problem 4.

Let $f_j : D(0,1) \to D(0,1)$ be holomorphic for each $j \in \mathbb{N}$. Suppose

$$\lim_{j\to\infty} f_j(0) = 1.$$

Show that $f_j(z) \to 1$ uniformly on compact subsets of $D(0,1)$.

## Problem 5.

Determine the number of zeros of the polynomial

$$z^{100} + 50z^{50} + 100z^2 + 1$$

in the annulus $\{1 < |z| < 2\}$.

## Problem 6.

Let $\mathcal{F}$ be the family of all analytic functions

$$f(z) = z + a_2z^2 + a_3z^3 + \dots$$

on the open unit disc, such that $|a_n| \le n$ for each $n$. Prove that $\mathcal{F}$ is a normal family.

## Problem 7.

Let $U$ be a bounded open connected set, $\{f_n\}$ a sequence of continuous functions on the closure of $U$, analytic in $U$. Assume that $\{f_n\}$ converges uniformly on $\partial U$. Prove that $\{f_n\}$ converges uniformly on $U$.