Complex Analysis, HW # 3

Section 7, problem 65, **and these problems:**

Problem 1.

Suppose that f is a nonconstant entire function and f is *odd*. Show that $f(\mathbb{C}) = \mathbb{C}$.

Problem 2.

Give an example of a nonconstant *even* entire function with $f(\mathbb{C}) \neq \mathbb{C}$.

Problem 3.

Here is a "counterexample" to the Little Picard Theorem:

The function e^z is entire and does not take value 0, therefore the function e^{e^z} is an entire function that does not take values 0 and 1!

What is the problem with this "counterexample"?

Problem 4.

Suppose that f is a nonconstant entire function which is not a polynomial. Prove that f assumes every value in $\mathbb{C}$ infinitely many times with at most one exception.
(Hint: use the Big Picard Theorem.)

Problem 5.

Suppose f and g are entire functions and $e^f + e^g \equiv 1$. Prove that f and g are constant functions.

Problem 6.

Suppose f and g are entire functions. Prove that the function $e^f + e^g$ either has no zeros or infinitely many zeros.