COMPLEX ANALYSIS MATH 220C

Midterm Sample Exam

Problem 1.

Let S be a sequence of points in the complex plane that converges to 0. Let $f(z)$ be defined and analytic on some disc centered at 0 except possibly at the points of S and at 0. Show that either $f(z)$ extends to be meromorphic in some disc containing 0, or else for any complex number w there is a sequence $\{\xi_j\}$ such that $\xi_j \to 0$ and $f(\xi_j) \to w$ as $j \to \infty$.

Problem 2.

Prove that if $f, g \in Aut(\mathbb{D})$ commute (that is, $f \circ g = g \circ f$) then they must be of the same type (i.e. both hyperbolic, or both elliptic, or both parabolic).

Problem 3.

a) Prove that a zero-free entire function of finite order whose derivative is also zero-free must be of order one.

b) Give an explicit example of such entire function.

c) Describe all such functions explicitly.

Problem 4.

Let $U \subset \mathbb{R}^2 = \mathbb{C}$ be open and convex. As with functions on the real line, one calls a function $\phi : U \to \mathbb{R}$ of two real variables convex if $\phi\left(\frac{z+w}{2}\right) \leq \frac{\phi(z)+\phi(w)}{2}$ for all $z, w \in U$. One can show (take it for granted) that measurable convex functions are automatically continuous. Given this, show that a convex function is subharmonic. Show by example that a subharmonic function need not be convex.

Problem 5.

Under what conditions on a sequence of real numbers $\{y_n\}_{n\in\mathbb{N}}$ does there exists a non-trivial bounded holomorphic function in the open right half-plane which has a zero at each point $1 + iy_n$? In particular, can this happen if

a) $y_n = \log n$?
b) $y_n = \sqrt{n}$?
c) $y_n = n$?
d) $y_n = n^2$?